Satisfiability Modulo Theories: A Calculus of Computation

PUC, Rio de Janeiro, 2009

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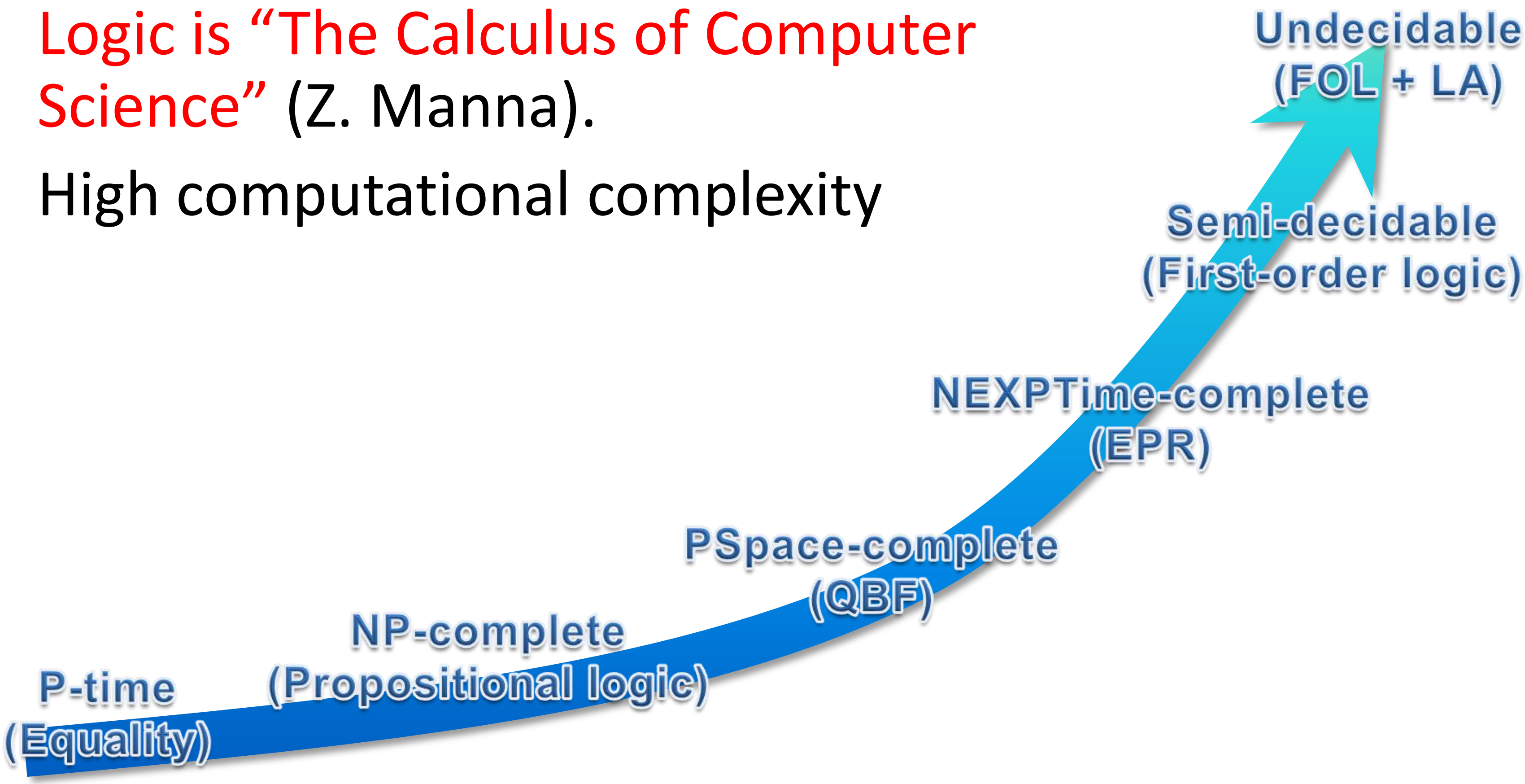
Symbolic Reasoning

Verification/Analysis tools
need some form of
Symbolic Reasoning

Symbolic Reasoning

- Logic is "The Calculus of Computer Science" (Z. Manna).
- High computational complexity



Applications

- Test case generation
- Verifying Compilers
- Predicate Abstraction
- Invariant Generation
- Type Checking
- Model Based Testing

Some Applications @ Microsoft

The Spec# Programming System

HAVOC

FormuLa

Hyper-V Microsoft Virtualization

Terminator T-2

VCC

SLAM

Yogi

NModel

Vigilante

SpecExplorer

Pex

F7

SAGE

Test case generation

```
unsigned GCD(x, y) {
  requires(y > 0);
  while (true) {
     unsigned m = x % y;
     if (m == 0) return y;
     x = y;
     y = m;
  }
}
```

SSA

$(y_0 > 0)$ and

$(m_0 = x_0 \% y_0)$ and

not $(m_0 = 0)$ and

$(x_1 = y_0)$ and

$(y_1 = m_0)$ and

$(m_1 = x_1 \% y_1)$ and

$(m_1 = 0)$

Solver

$x_0 = 2$

$y_0 = 4$

$m_0 = 2$

$x_1 = 4$

$y_1 = 2$

$m_1 = 0$

We want a trace where the loop is executed twice.

Type checking

Signature:

div : int, $\{ x : \text{int} \mid x \neq 0 \} \rightarrow \text{int}$

Subtype

Call site:

if a $\leq$ 1 and a $\leq$ b then

return div(a, b)

Verification condition

a $\leq$ 1 and a $\leq$ b implies b $\neq$ 0

Satisfiability Modulo Theories (SMT)

Is formula *F* satisfiable modulo theory *T* ?

SMT solvers have specialized algorithms for *T*

Satisfiability Modulo Theories (SMT)

$$b + 2 = c \ \ and \ \ f(read(write(a,b,3), c\text{-}2) \neq f(c\text{-}b+1)$$

Satisfiability Modulo Theories (SMT)

$\boxed{b + 2 = c}$ *and* $f(read(write(a,b,\boxed{3}), \boxed{c-2}) \neq f(\boxed{c-b+1})$

Arithmetic

Satisfiability Modulo Theories (SMT)

$$b + 2 = c \ \ and \ \ f(\boxed{read}(\boxed{write}(a,b,3), c\text{-}2) \neq f(c\text{-}b+1)$$

Array Theory

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Satisfiability Modulo Theories (SMT)

$$b + 2 = c \ \ and \ \ \boxed{f}(read(write(a,b,3), c\text{-}2) \neq \boxed{f}(c\text{-}b+1)$$

Uninterpreted Functions

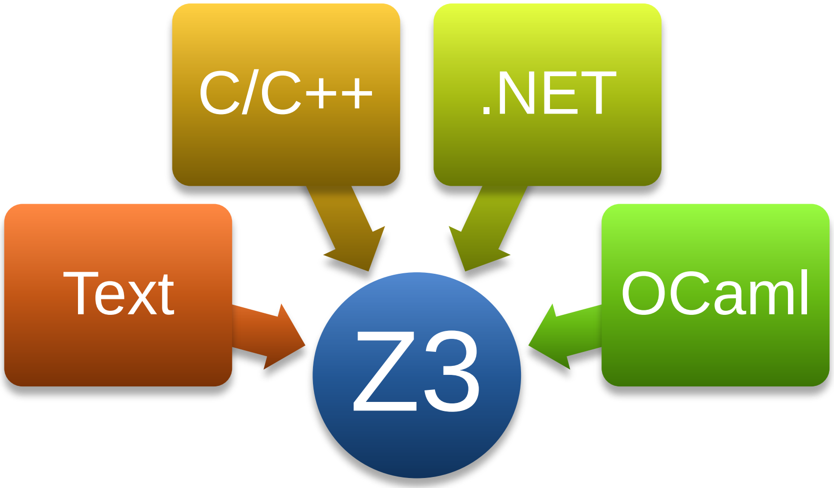
Theories

- ***A Theory is a set of sentences***

- Alternative definition:
  A Theory is a class of structures

SMT@Microsoft: Solver

- Z3 is a new solver developed at Microsoft Research.
- Development/Research driven by internal customers.
- Free for academic research.
- Interfaces:



- http://research.microsoft.com/projects/z3

Ground formulas

For most SMT solvers: ***F is a set of ground formulas***

Many Applications

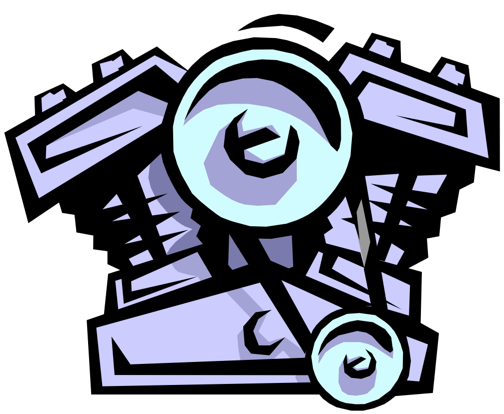
Bounded Model Checking

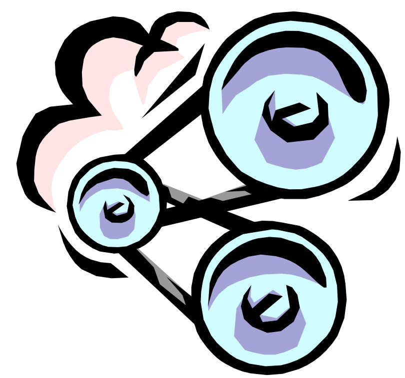
Test-Case Generation

Little Engines of Proof

An SMT Solver is a collection of

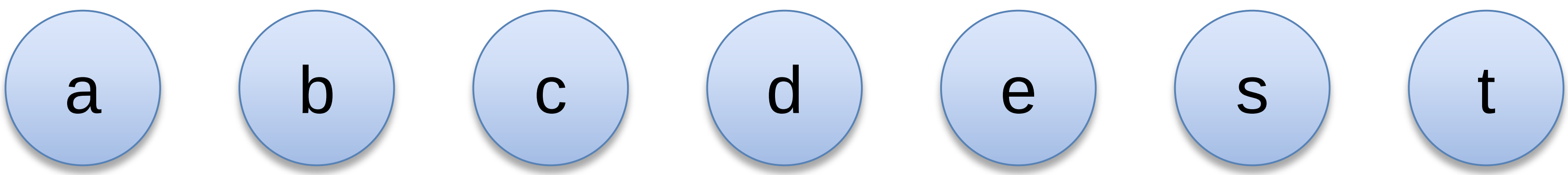
Little Engines of Proof





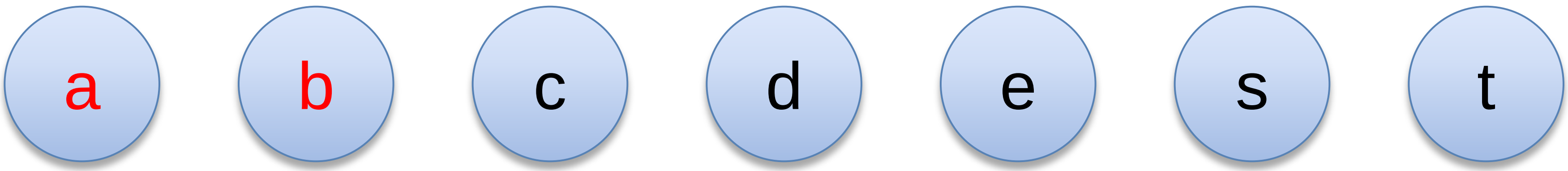
Deciding Equality

$$a = b,\ b = c,\ d = e,\ b = s,\ d = t,\ a \neq e,\ a \neq s$$



Deciding Equality

$a = b$, $b = c$, $d = e$, $b = s$, $d = t$, $a \neq e$, $a \neq s$



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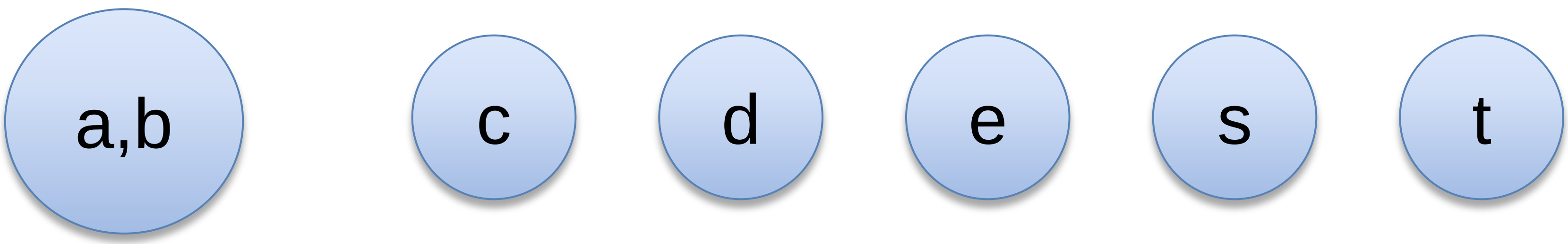
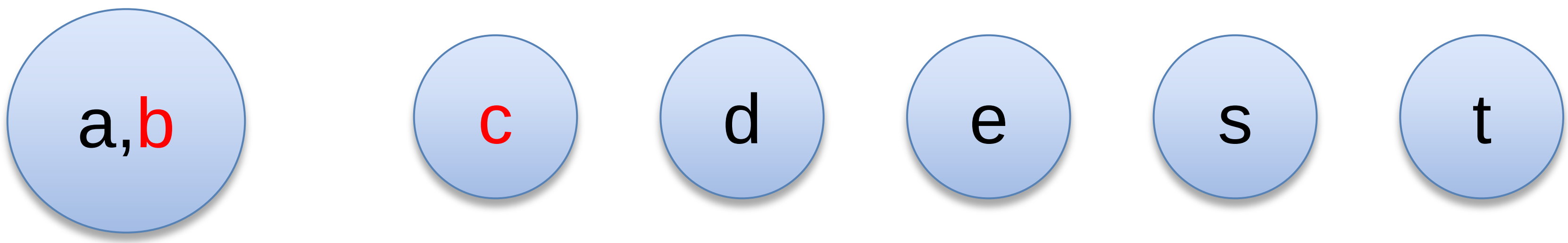


Deciding Equality

$$a = b,\ \color{red}{b = c},\ d = e,\ b = s,\ d = t,\ a \neq e,\ a \neq s$$



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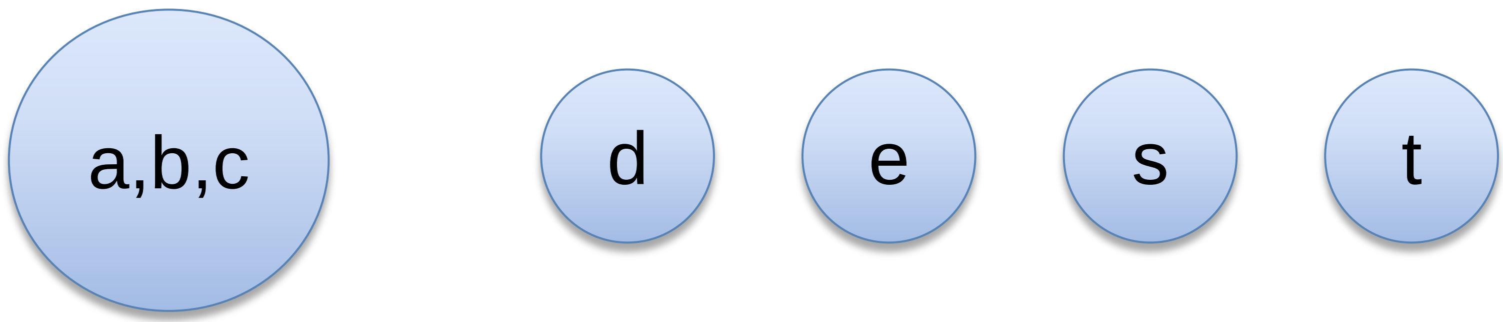

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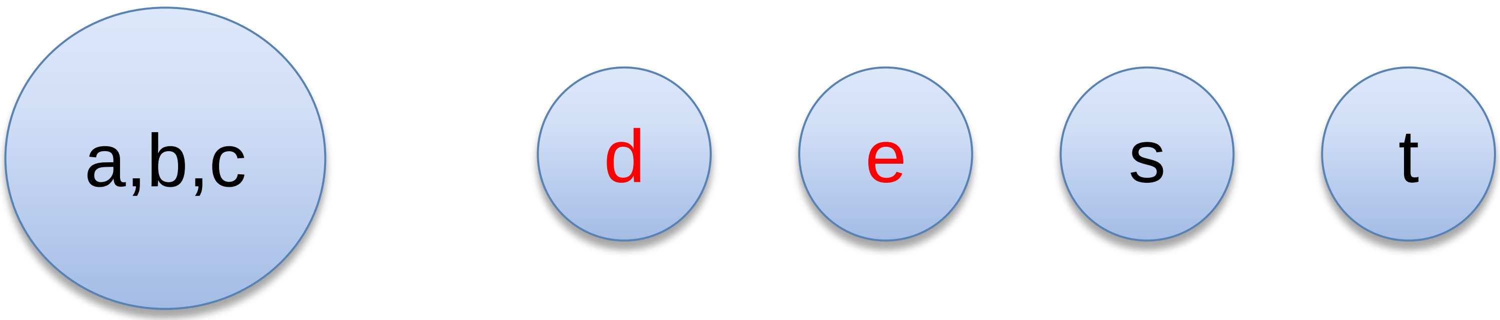

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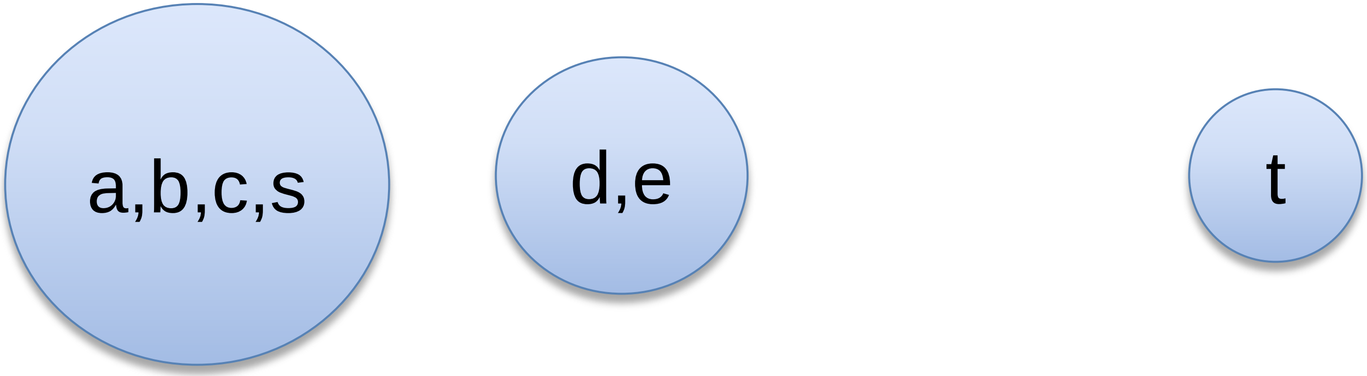

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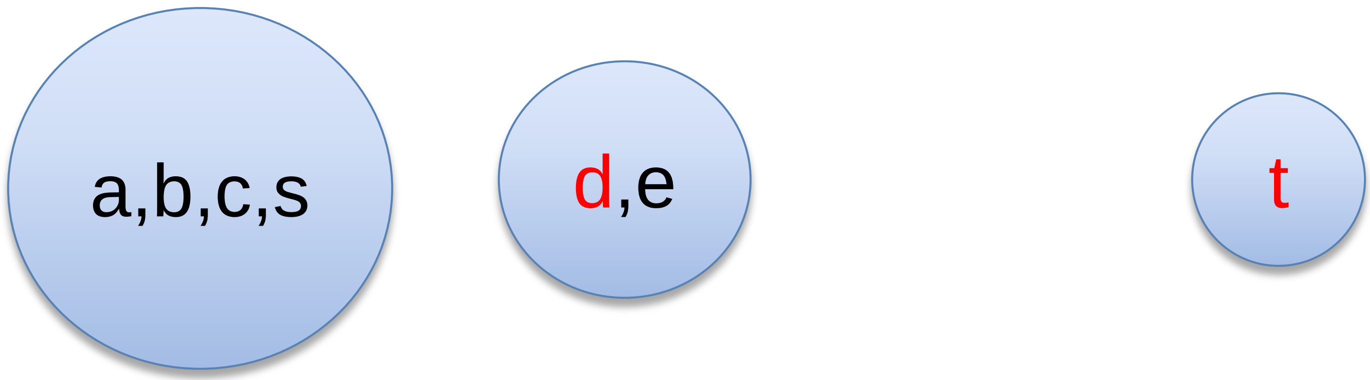

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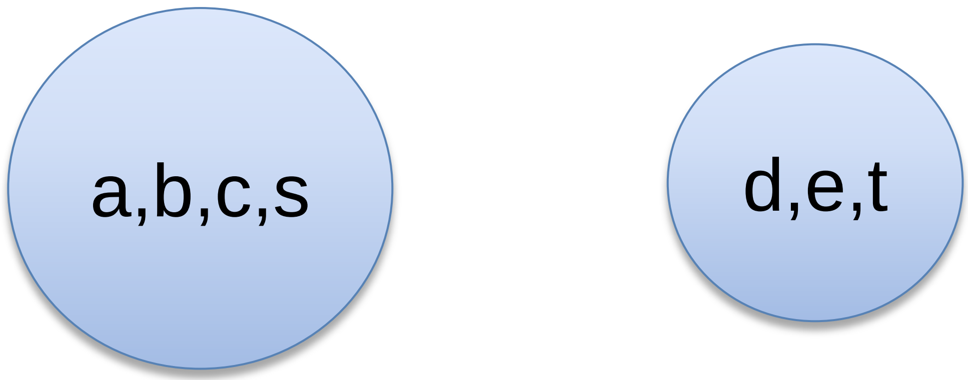

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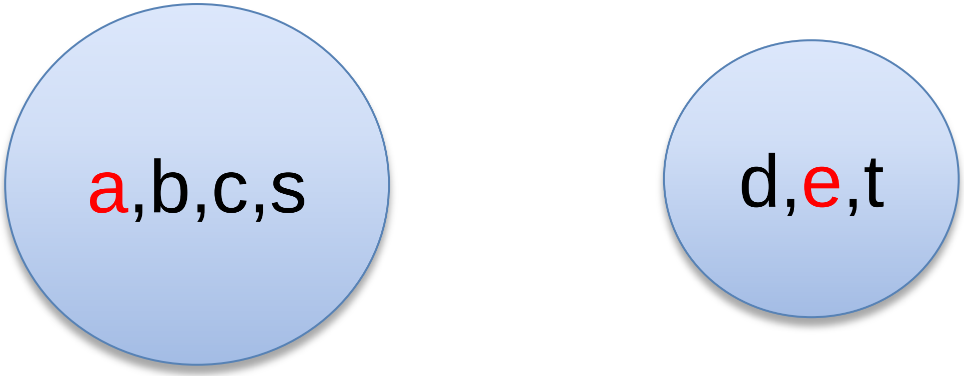

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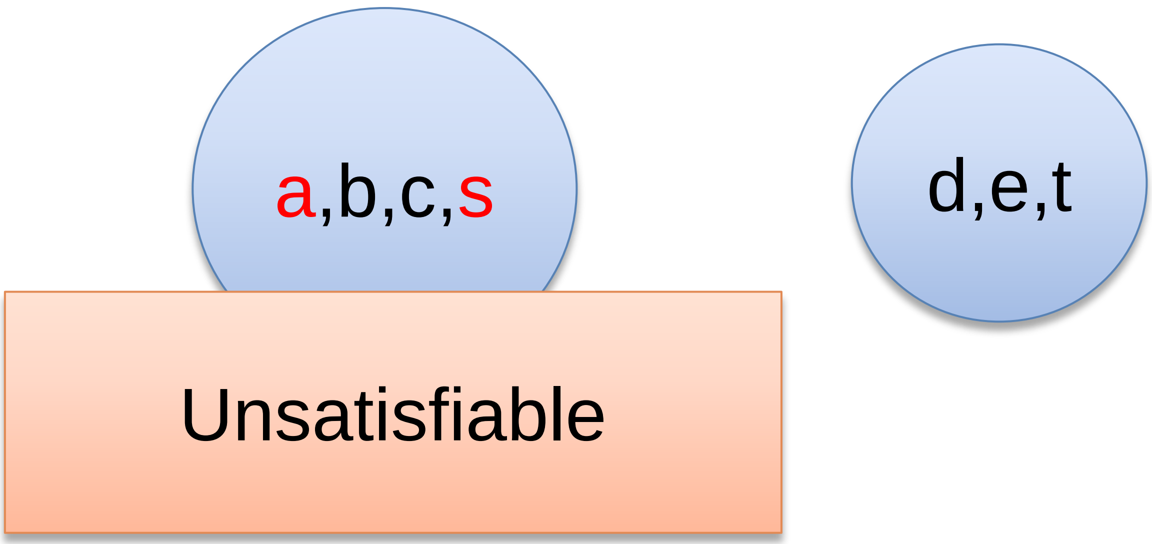

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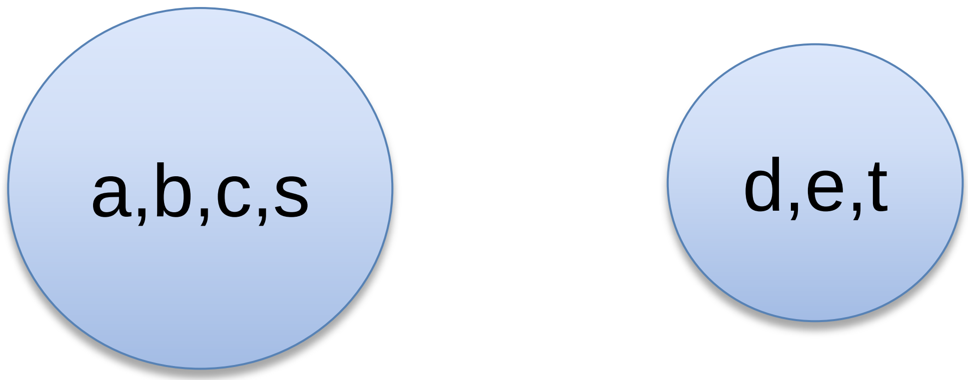


Model

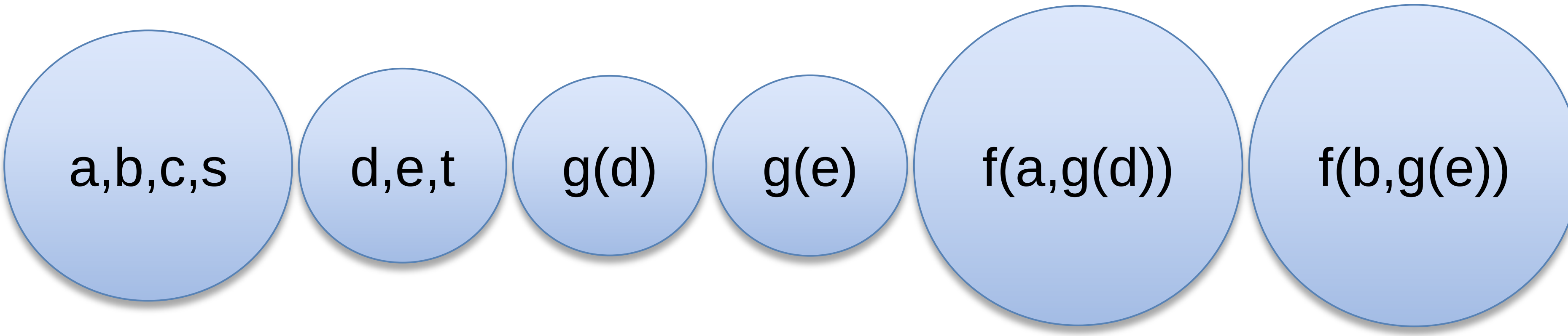
$|M| = \{\ 0, 1\ \}$

$M(a) = M(b) = M(c) = M(s) = 0$

$M(d) = M(e) = M(t) = 1$

Deciding Equality + (uninterpreted) Functions

$a = b, b = c, d = e, b = s, d = t, f(a, g(d)) \neq f(b, g(e))$



Congruence Rule:

$x_1 = y_1, \ldots, x_n = y_n$ implies $f(x_1, \ldots, x_n) = f(y_1, \ldots, y_n)$

Deciding Equality + (uninterpreted) Functions

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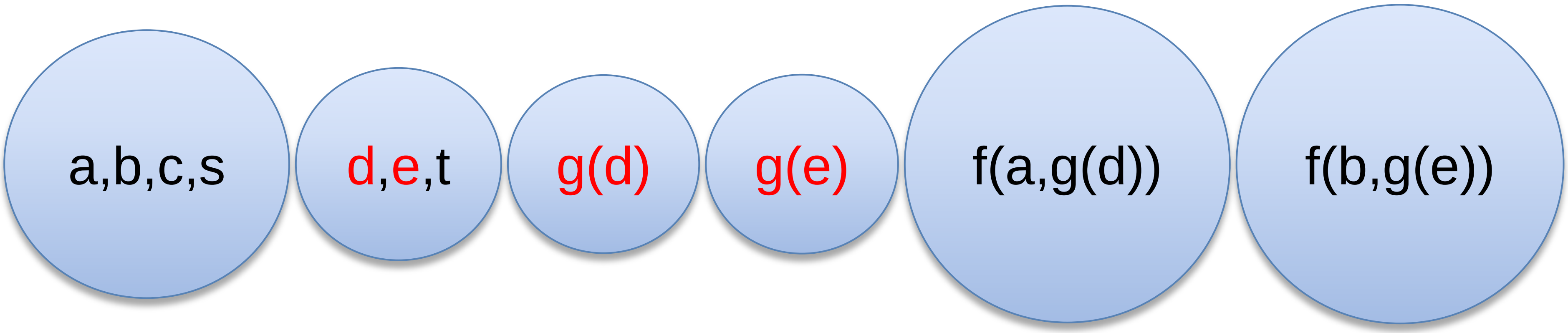

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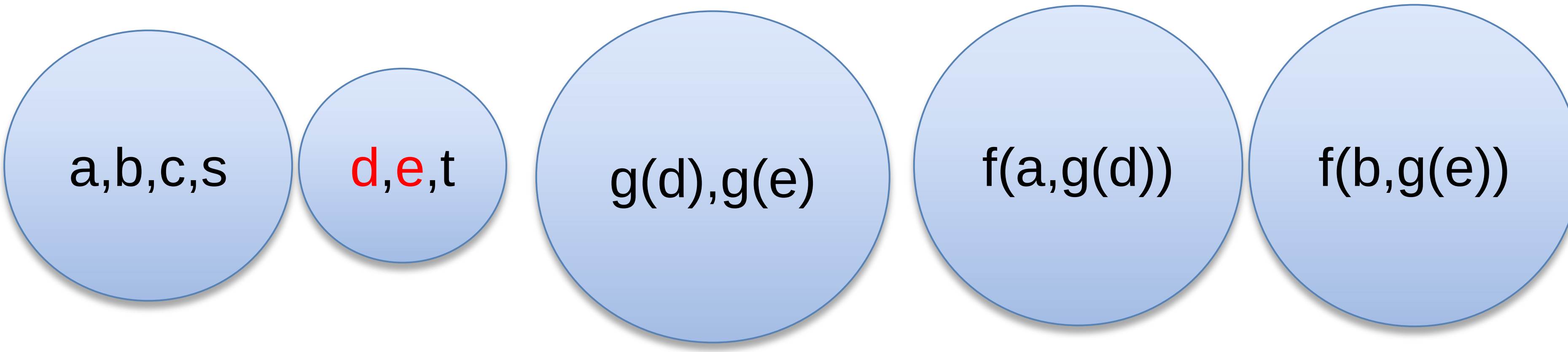

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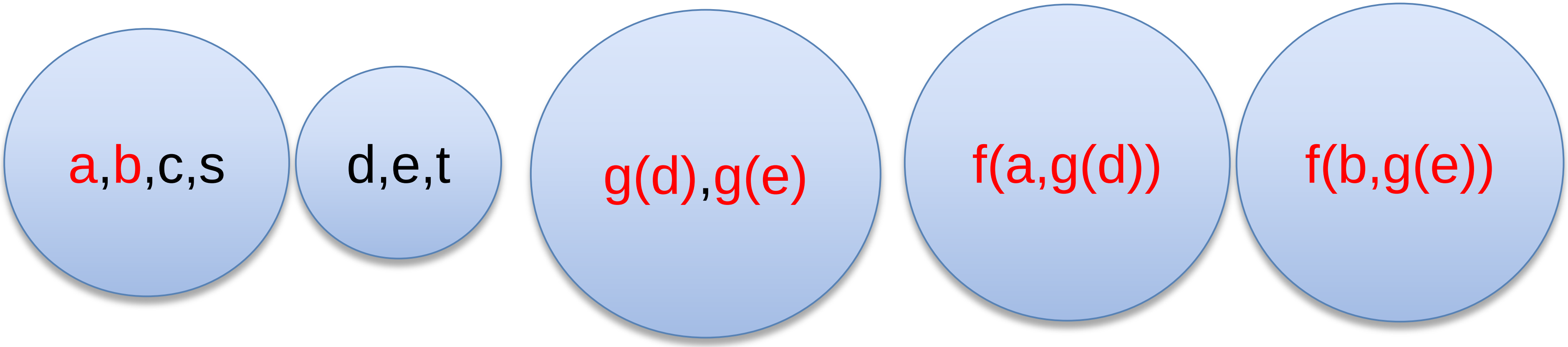

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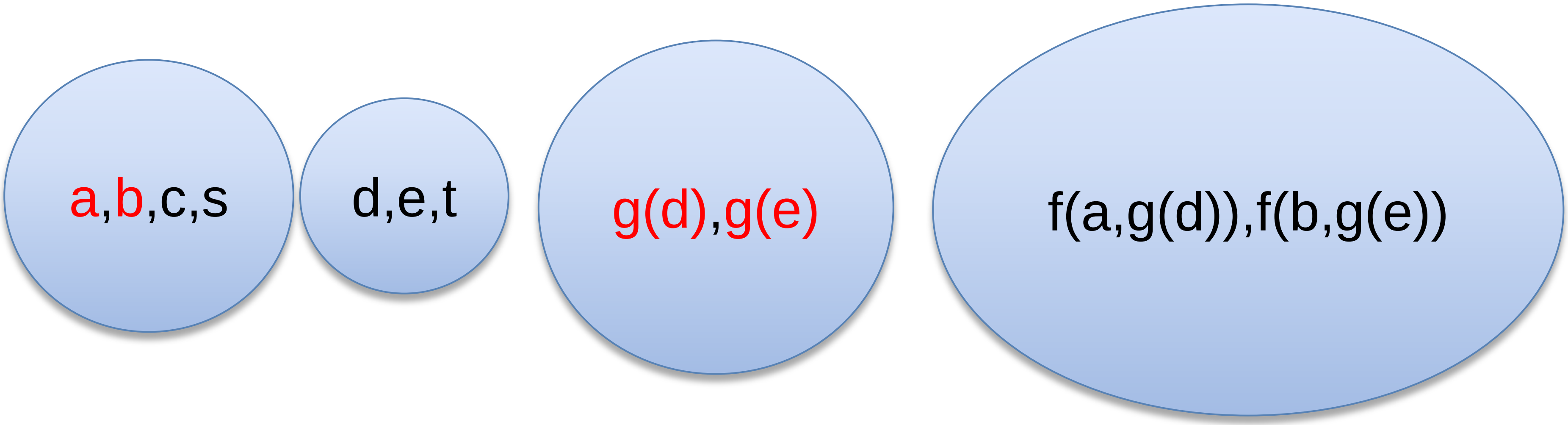

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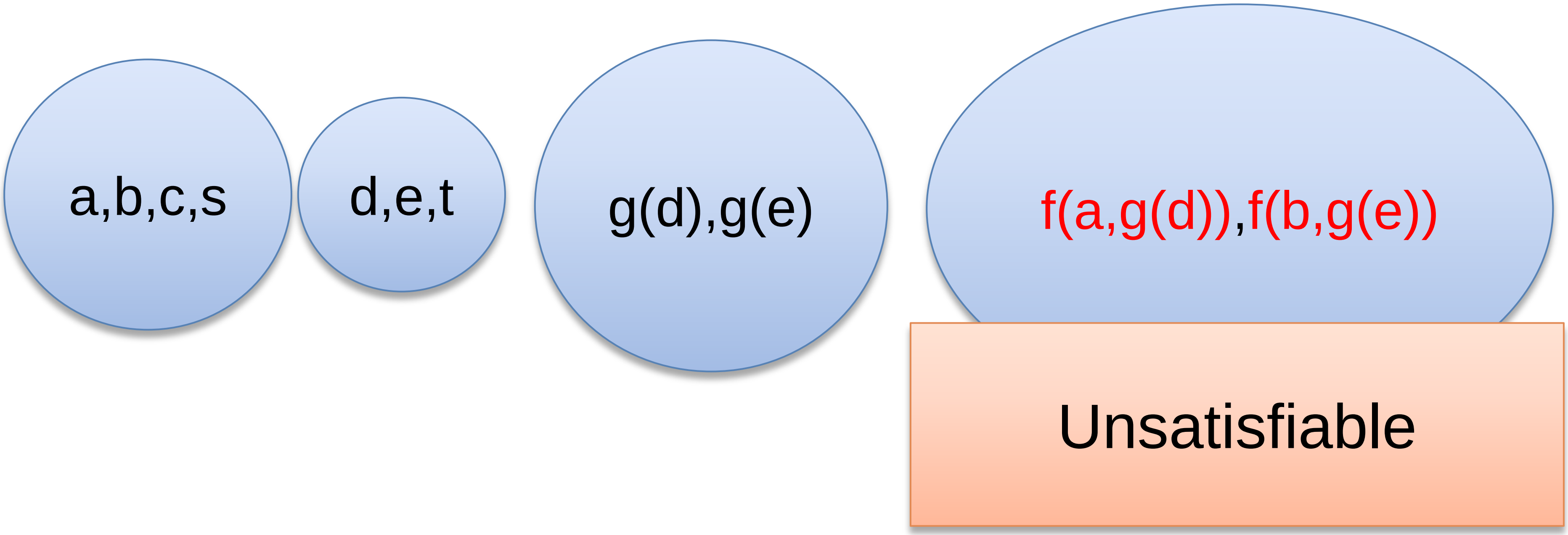


Deciding Equality + (uninterpreted) Functions

(fully shared) DAGs for representing terms

Union-find data-structure + Congruence Closure

O(n log n)

Deciding Equality + (uninterpreted) Functions

Many theories can be reduced to
Equality + Uninterpreted functions

Arrays, Sets
Lists, Tuples
Inductive Datatypes

Deciding Difference Arithmetic

$$a - b \leq 2$$

$$x := x + 1$$

$$\Downarrow$$

$$x_1 = x_0 + 1$$

$$\Downarrow$$

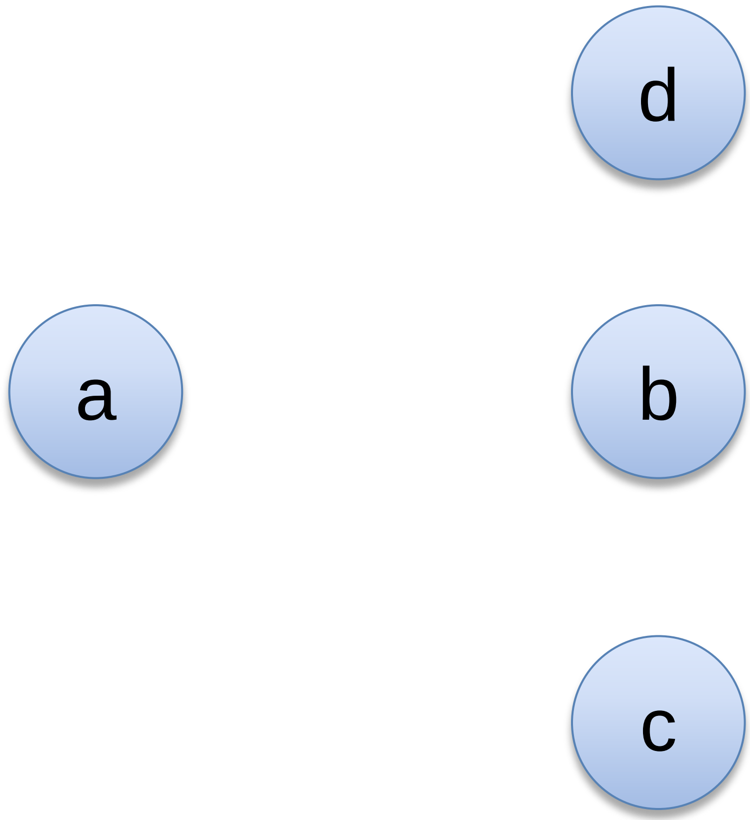
$$x_1 \leq x_0 + 1, \;\; x_1 \geq x_0 + 1$$

$$\Downarrow$$

$$x_1 - x_0 \leq 1, \;\; x_0 - x_1 \leq -1$$

Deciding Difference Arithmetic

$a - b \leq 2, \quad b - c \leq -3, \quad b - d \leq -1, \quad d - a \leq 4, \quad c - a \leq 0$



Deciding Difference Arithmetic

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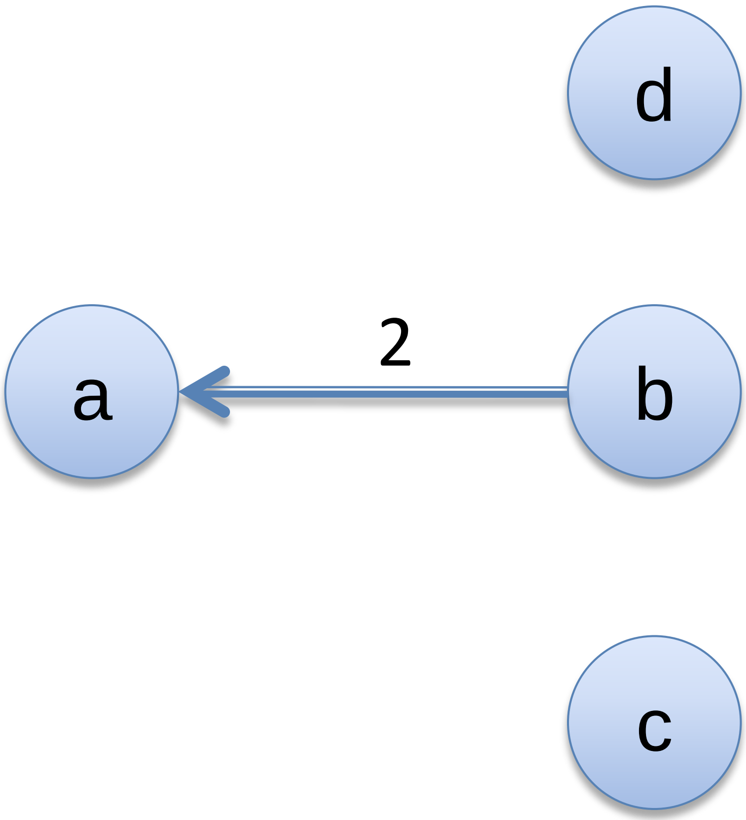

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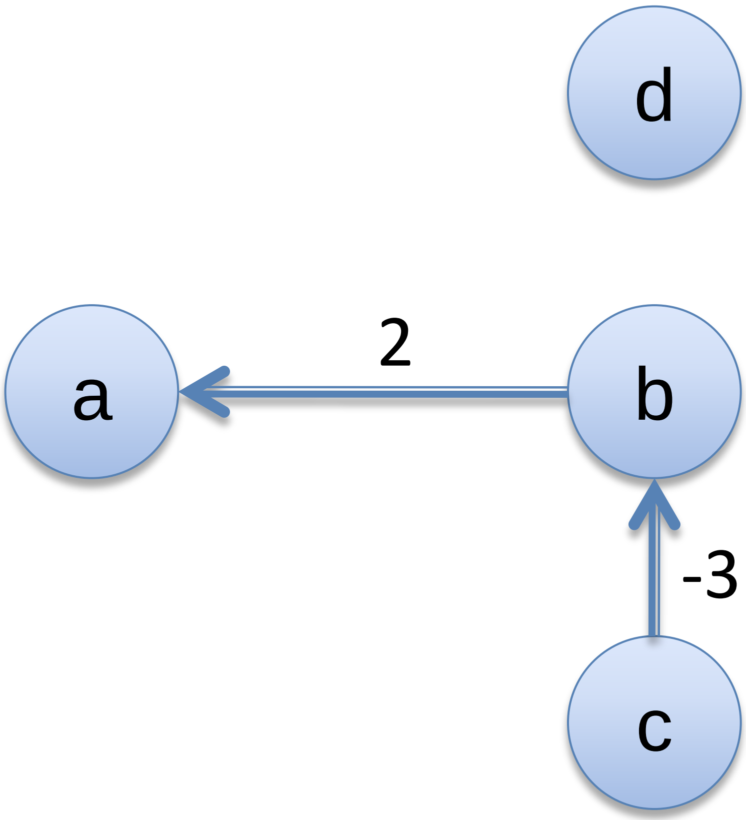

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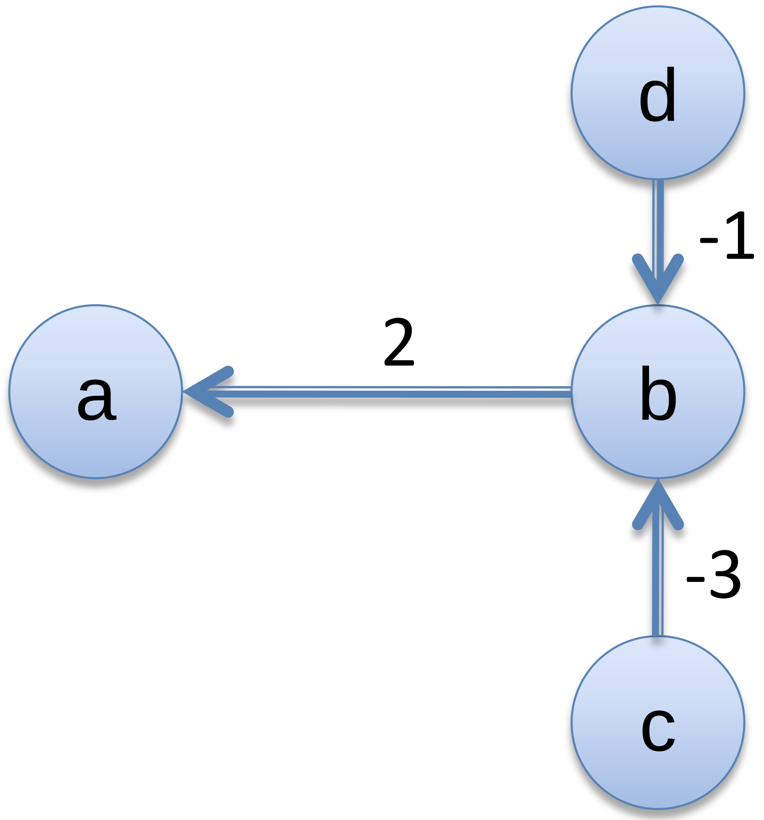
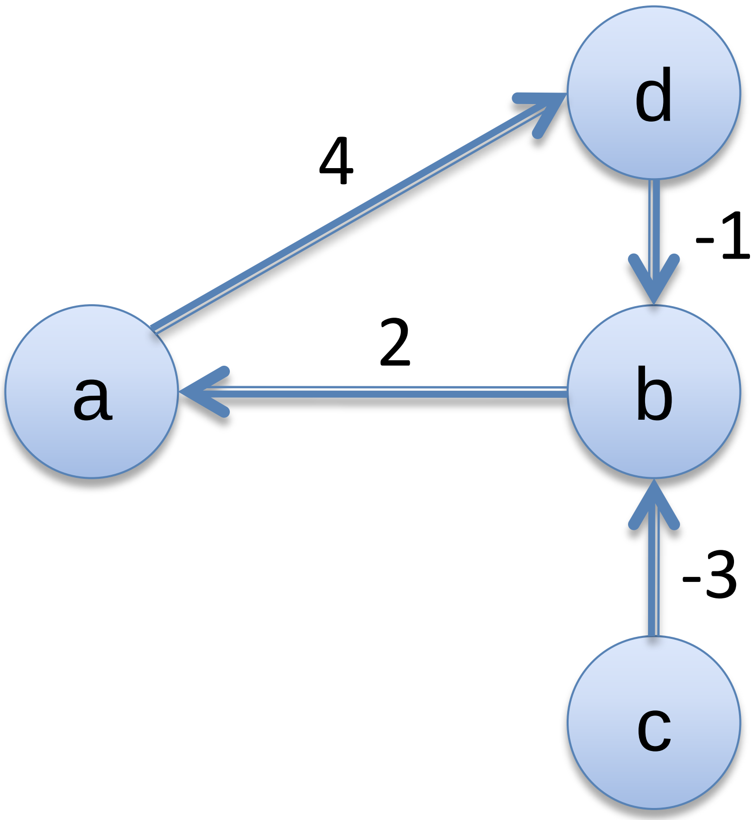


Deciding Difference Arithmetic

$a - b \leq 2, \quad b - c \leq -3, \quad b - d \leq -1, \quad {\color{red} d - a \leq 4}, \quad c - a \leq 0$



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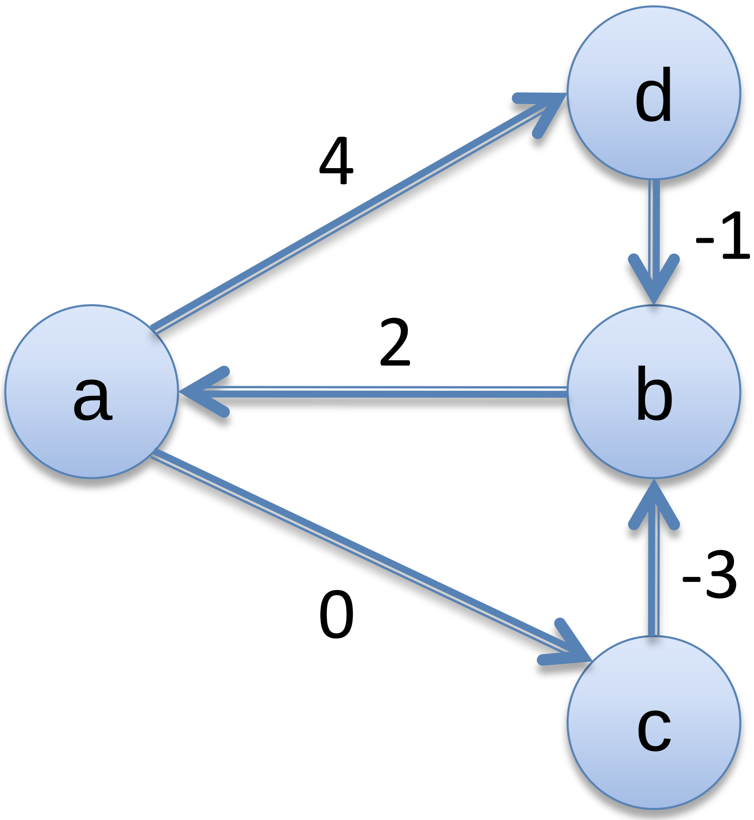
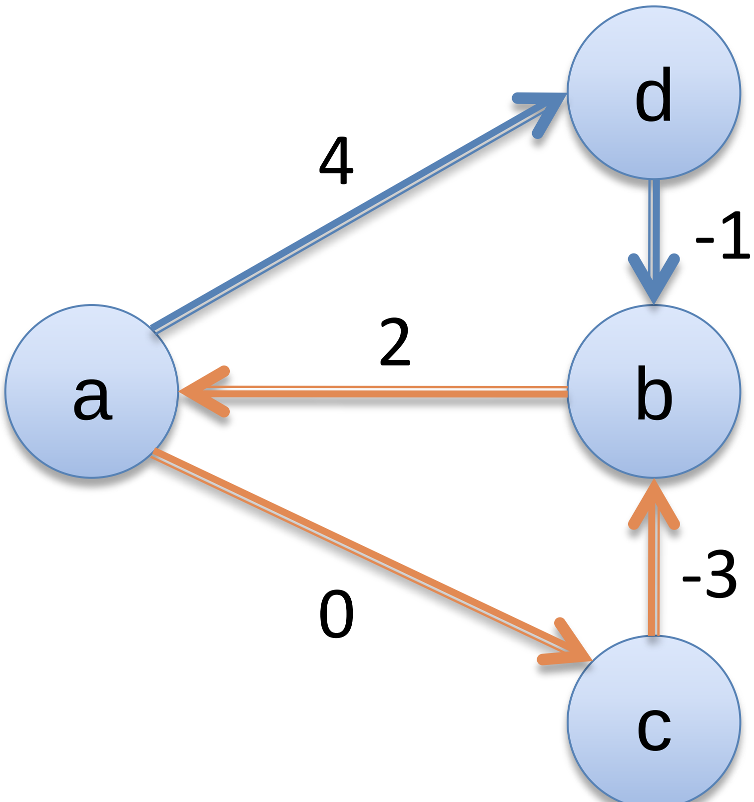


Deciding Difference Arithmetic

$a - b \leq 2, \quad b - c \leq -3, \quad b - d \leq -1, \quad d - a \leq 4, \quad c - a \leq 0$

$a - b \leq 2,$
$b - c \leq -3,$
$c - a \leq 0$

$0 \leq -1$



Unsatisfiable
iff
Negative Cycle

Deciding Difference Arithmetic

Shortest Path Algorithms

Bellman-Ford: $O(nm)$

Floyd-Warshall: $O(n^3)$

Other Little Engines

Linear Arithmetic	Simplex, Fourier Motzkin
Non-linear (complex) arithmetic	Grobner Basis
Non-linear (real) arithmetic	Cylindrical Algebraic Decomposition
Equational theories	Knuth-Bendix Completion
Bitvectors	Bit-blasting, rewriting

Combining Solvers

In practice, we need a combination of theory solvers.

Nelson-Oppen combination method.

Reduction techniques.

Model-based theory combination.

SAT (propositional checkers): Case Analysis

$$
\begin{array}{rl}
p \lor & q, \\
p \lor & \neg q, \\
\neg p \lor & q, \\
\neg p \lor & \neg q
\end{array}
$$

SAT (propositional checkers): Case Analysis

$p \lor q$,
$p \lor \neg q$,
$\neg p \lor q$,
$\neg p \lor \neg q$

Assignment:
p = false,
q = false

SAT (propositional checkers): Case Analysis

$p \lor q$,

$p \lor \neg q$,

$\neg p \lor q$,

$\neg p \lor \neg q$

Assignment:

p = false,

q = true

SAT (propositional checkers): Case Analysis

$p \vee q$,
$p \vee \neg q$,
$\neg p \vee q$,
$\neg p \vee \neg q$

Assignment:
p = true,
q = false

SAT (propositional checkers): Case Analysis

$$p \lor q,$$
$$p \lor \neg q,$$
$$\neg p \lor q,$$
$$\neg p \lor \neg q$$

Assignment:
p = true,
q = true

SAT (propositional checkers): DPLL

M | F

Partial model

Set of clauses

DPLL

- Guessing (case-splitting)

$$p \mid p \lor q, \neg q \lor r$$

$$\Downarrow$$

$$p, \neg q \mid p \lor q, \neg q \lor r$$

DPLL

- Deducing

$$p \mid p \vee q, \neg p \vee s$$

$$\Downarrow$$

$$p, s \mid p \vee q, \neg p \vee s$$

DPLL

- Backtracking

$p, \neg s, q \mid p \vee q, s \vee q, \neg p \vee \neg q$

$p, s \mid p \vee q, s \vee q, \neg p \vee \neg q$

Modern DPLL

- Efficient indexing (two-watch literal)
- Non-chronological backtracking (backjumping)
- Lemma learning
- …

Solvers = DPLL + Decision Procedures

- Efficient decision procedures for conjunctions of ground literals.

$$a=b,\ a<5 \mid \neg a=b \vee f(a)=f(b),\quad a < 5 \vee a > 10$$

Theory Conflicts

$a=b,\ a > 0,\ c > 0,\ a + c < 0 \mid F$

backtrack

Naïve recipe?

SMT Solver = DPLL + Decision Procedure

Standard question:

Why don't you use CPLEX for handling linear arithmetic?

Efficient SMT solvers

Decision Procedures must be:

Incremental & Backtracking

Theory Propagation

$a=b,\ a<5 \mid \ldots\ a<6 \vee f(a) = a$

$\Downarrow$

$a=b,\ a<5,\ a<6 \mid \ldots\ a<6 \vee f(a) = a$

Efficient SMT solvers

Decision Procedures must be:

Incremental & Backtracking

Theory Propagation

Precise (theory) lemma learning

$a=b, a > 0, c > 0, a + c < 0 \mid F$

Learn clause:

$\neg(a=b) \vee \neg(a > 0) \vee \neg(c > 0) \vee \neg(a + c < 0)$

Imprecise!

Precise clause:

$\neg a > 0 \vee \neg c > 0 \vee \neg a + c < 0$

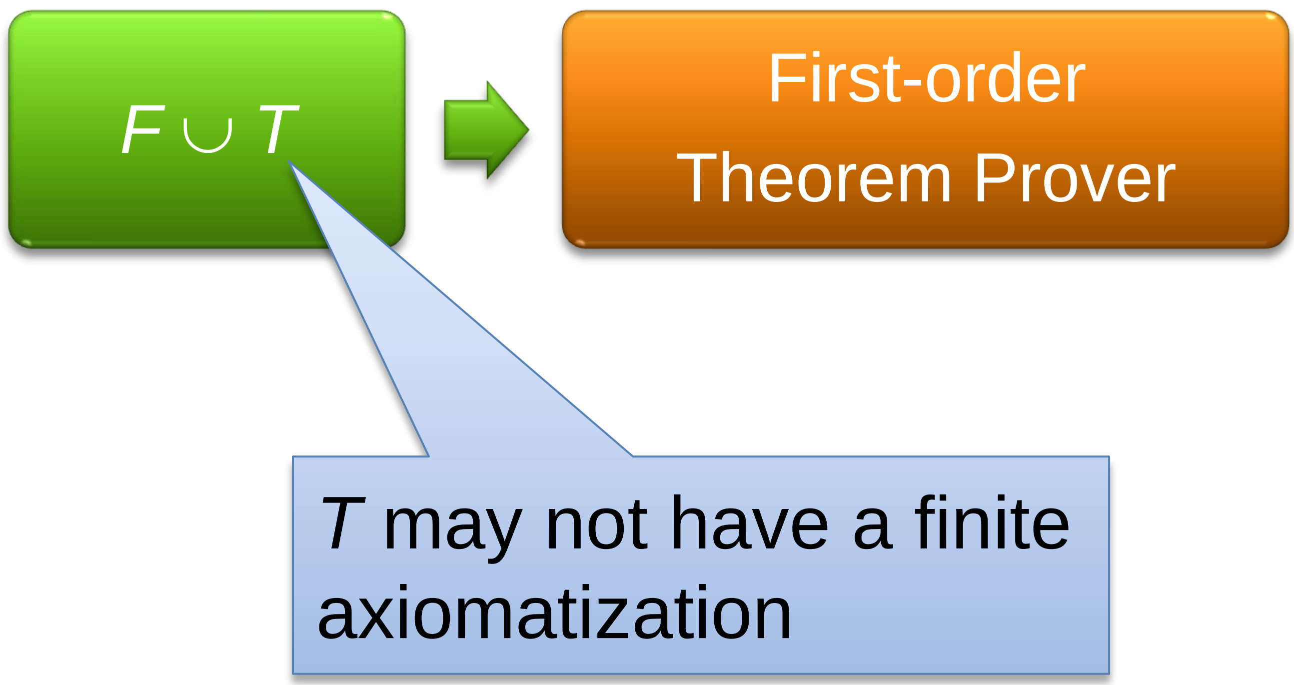
SMT x SAT

For some theories, SMT can be reduced to SAT

Higher level of abstraction

$$\mathbf{bvmul}_{32}(a,b) = \mathbf{bvmul}_{32}(b,a)$$

SMT x First-order provers



Test case generation

Test case generation

- Test (correctness + usability) is 95% of the deal:
  - Dev/Test is 1-1 in products.
  - Developers are responsible for unit tests.
- Tools:
  - Annotations and static analysis (SAL + ESP)
  - File Fuzzing
  - Unit test case generation

Security is critical

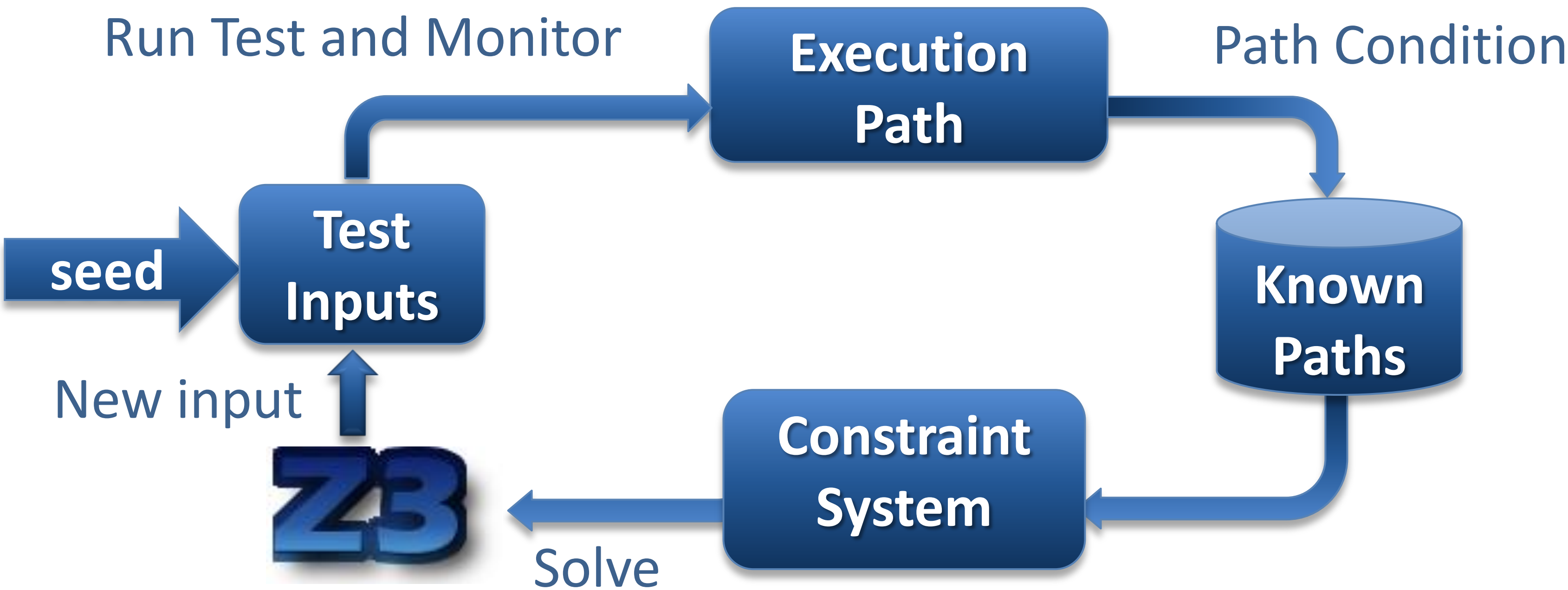
- Security bugs can be very expensive:
  - Cost of each MS Security Bulletin: $600k to $Millions.
  - Cost due to worms: $Billions.
  - The real victim is the customer.
- Most security exploits are initiated via files or packets.
  - Ex: Internet Explorer parses dozens of file formats.
- Security testing: hunting for million dollar bugs
  - Write A/V
  - Read A/V
  - Null pointer dereference
  - Division by zero

Hunting for Security Bugs.

- Two main techniques used by *"black hats"*:
  - Code inspection (of binaries).
  - *Black box fuzz testing.*

- **Black box** fuzz testing:
  - A form of black box random testing.
  - Randomly *fuzz* (=modify) a well formed input.
  - Grammar-based fuzzing: rules to encode how to fuzz.
- **Heavily** used in security testing
  - At MS: several internal tools.
  - Conceptually simple yet effective in practice

Directed Automated Random Testing (DART)



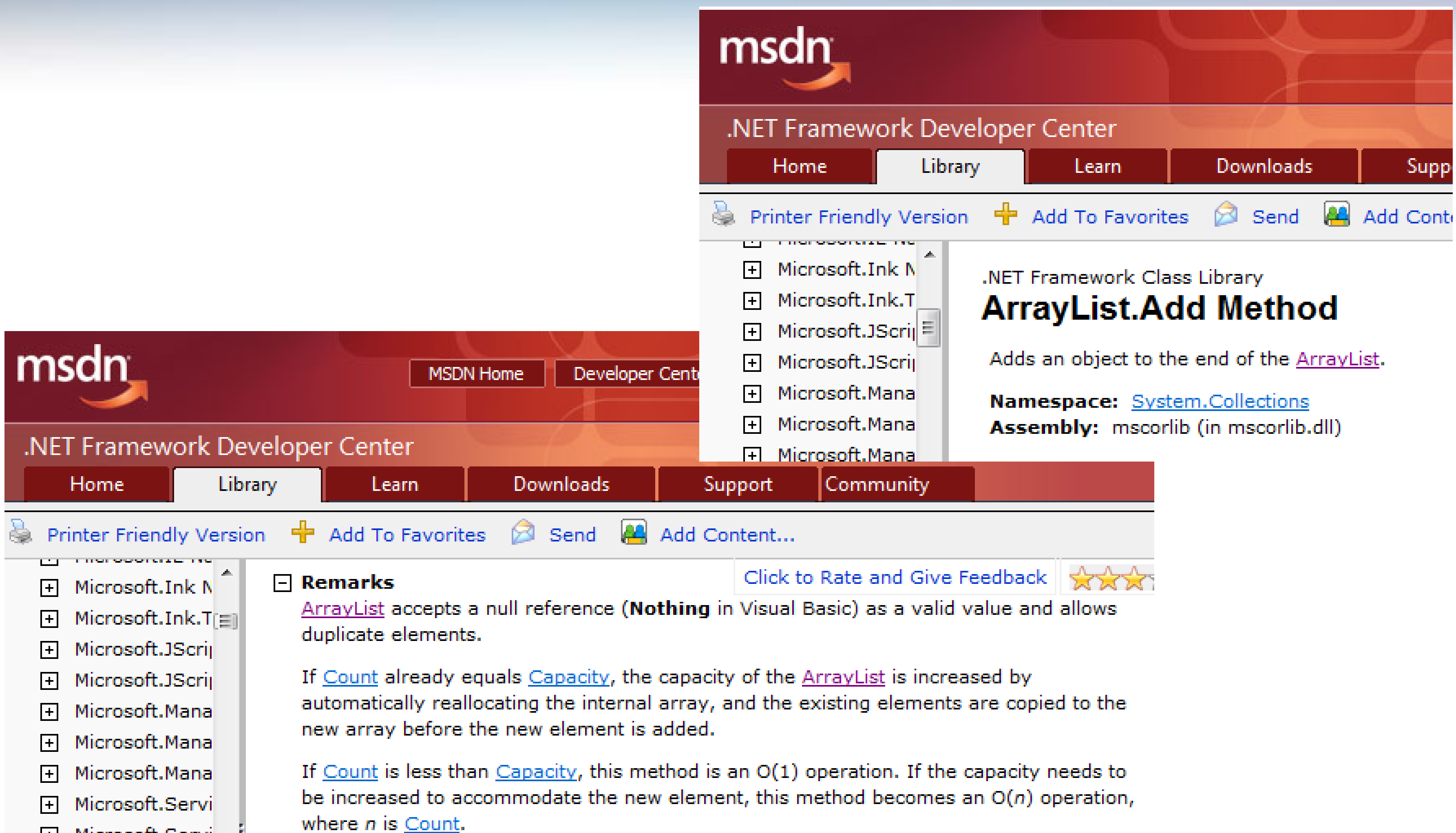
DARTish projects at Microsoft

PEX	Implements DART for .NET.
SAGE	Implements DART for x86 binaries.
YOGI	Implements DART to check the feasibility of program paths generated statically.
Vigilante	Partially implements DART to dynamically generate worm filters.

What is Pex?

- Test input generator
  - Pex starts from parameterized unit tests
  - Generated tests are emitted as traditional unit tests

ArrayList: The Spec



ArrayList: AddItem Test

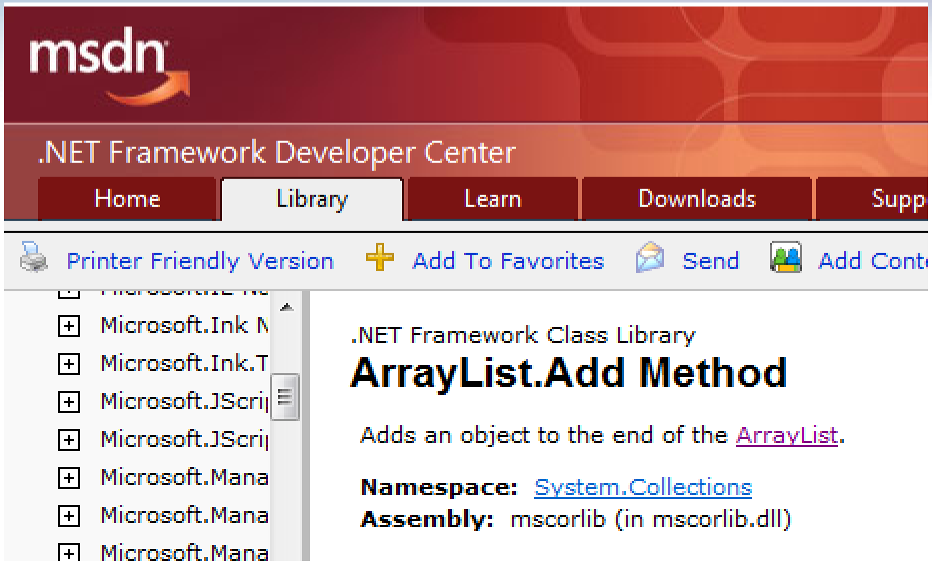
```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```



Microsoft Research

ArrayList: Starting Pex…

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Inputs

Microsoft Research

ArrayList: Run 1, (0,null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Inputs
(0,null)

Microsoft Research

ArrayList: Run 1, (0,null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

	Inputs	Observed Constraints
	(0,null)	!(c<0)

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

c < 0 → false

Microsoft Research

ArrayList: Run 1, (0,null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

0 == c → true

Microsoft® Research

ArrayList: Run 1, (0,null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
item == item  →  true
```

	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Microsoft Research

ArrayList: Picking the next branch to cover

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && **0!=c**		

ArrayList: Solve constraints using SMT solver

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && 0!=c	**(1,null)**	

ArrayList: Run 2, (1, null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

0 == c → false

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && 0!=c	(1,null)	!(c<0) && 0!=c

Microsoft Research

ArrayList: Pick new branch

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	`(0,null)`	`!(c<0) && 0==c`
`!(c<0) && 0!=c`	`(1,null)`	`!(c<0) && 0!=c`
`c<0`		

Z3

ArrayList: Run 3, (-1, null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	`(0,null)`	`!(c<0) && 0==c`
`!(c<0) && 0!=c`	`(1,null)`	`!(c<0) && 0!=c`
`c<0`	**`(-1,null)`**	

ArrayList: Run 3, (-1, null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && 0!=c	(1,null)	!(c<0) && 0!=c
c<0	**(-1,null)**	c<0

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

c < 0 → true

ArrayList: Run 3, (-1, null)

```
class ArrayListTest {
  [PexMethod]
  void AddItem(int c, object item) {
      var list = new ArrayList(c);
      list.Add(item);
      Assert(list[0] == item); }
}
```

```
class ArrayList {
  object[] items;
  int count;

  ArrayList(int capacity) {
    if (capacity < 0) throw ...;
    items = new object[capacity];
  }

  void Add(object item) {
    if (count == items.Length)
      ResizeArray();

    items[this.count++] = item; }
...
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && 0!=c	(1,null)	!(c<0) && 0!=c
c<0	(-1,null)	c<0

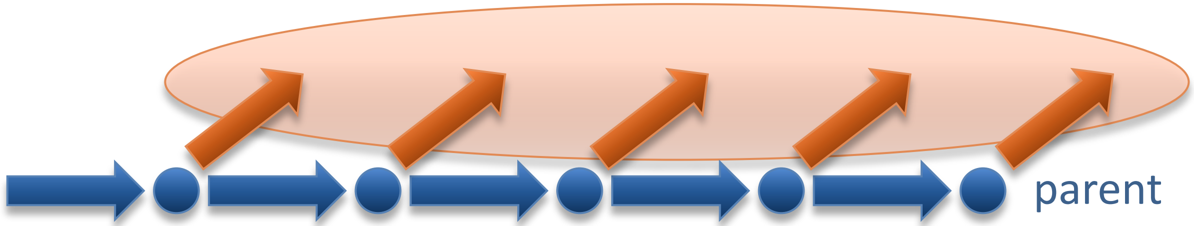
Microsoft Research

PEX ⟷ Z3

Rich Combination	Linear arithmetic → Bitvector → Arrays → Free Functions
Models	Model used as test inputs
∀-Quantifier	Used to model custom theories (e.g., .NET type system)
API	Huge number of small problems. Textual interface is too inefficient.

SAGE

- Apply DART to large applications (not units).
- Start with well-formed input (not random).
- Combine with generational search (not DFS).
  - Negate 1-by-1 each constraint in a path constraint.
  - Generate many children for each parent run.



Microsoft Research

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 0 – seed file

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 00 00 00 00 00 00 00 00 00 00 00 00 ; RIFF............
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 1

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 00 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF....***....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 2

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 3

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 00 00 00 00 ; ....strh........
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 4

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 5

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 00 00 00 00 00 00 00 00 ; ....strf........
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 6

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 00 00 00 00 28 00 00 00 ; ....strf....(...
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 7

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 00 00 00 00 28 00 00 00 ; ....strf....(...
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 C9 9D E4 4E ; ............ÉäN
00000060h: 00 00 00 00                                     ; ....
```

Generation 8

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 00 00 00 00 28 00 00 00 ; ....strf....(...
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 01 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 9

Zero to Crash in 10 Generations

- Starting with 100 zero bytes …
- SAGE generates a crashing test for Media1 parser

```
00000000h: 52 49 46 46 3D 00 00 00 ** ** ** 20 00 00 00 00 ; RIFF=...*** ....
00000010h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000020h: 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 ; ................
00000030h: 00 00 00 00 73 74 72 68 00 00 00 00 76 69 64 73 ; ....strh....vids
00000040h: 00 00 00 00 73 74 72 66 B2 75 76 3A 28 00 00 00 ; ....strf²uv:(...
00000050h: 00 00 00 00 00 00 00 00 00 00 00 00 01 00 00 00 ; ................
00000060h: 00 00 00 00                                     ; ....
```

Generation 10 – CRASH

SAGE (cont.)

- SAGE is very effective at finding bugs.
- Works on large applications.
- Fully automated
- Easy to deploy (x86 analysis – any language)
- Used in various groups inside Microsoft
- Powered by Z3.

SAGE ⟷ Z3

- Formulas are usually big conjunctions.
- SAGE uses only the bitvector and array theories.
- Pre-processing step has a huge performance impact.
  - Eliminate variables.
  - Simplify formulas.
- Early unsat detection.

Verifying Compilers

Annotated Program → Verification Condition *F*

pre/post conditions
invariants
and other annotations

Annotations: Example

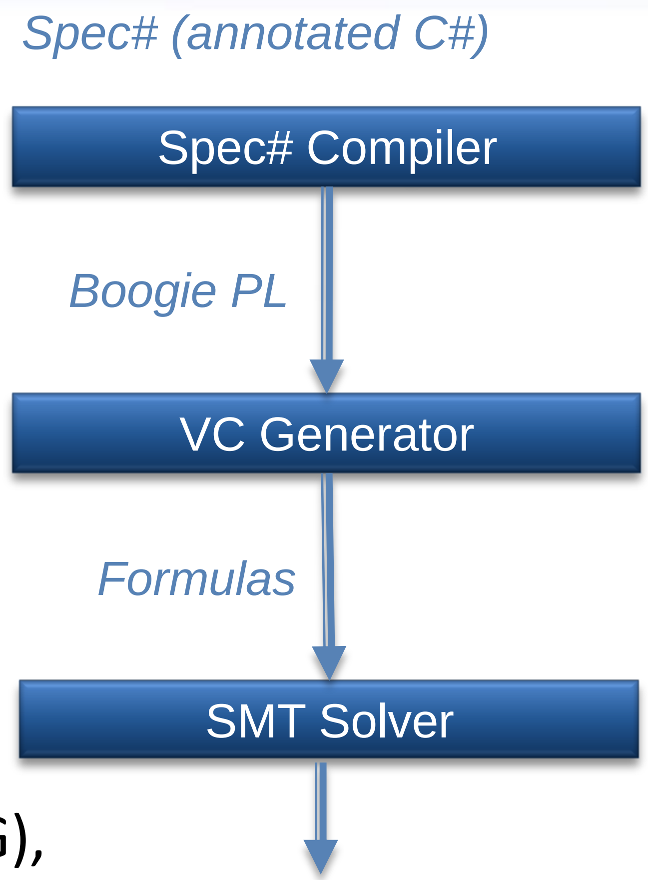
```
class C {
    private int a, z;
    invariant z > 0

    public void M()
        requires a != 0
        {
            z = 100/a;
        }
}
```

Microsoft Research

Spec# Approach for a Verifying Compiler

- *Source Language*
  - C# + goodies = Spec#
- *Specifications*
  - method contracts,
  - invariants,
  - field and type annotations.
- *Program Logic:*
  - *Dijkstra's weakest preconditions.*
- *Automatic Verification*
  - type checking,
  - verification condition generation (VCG),
  - **SMT**



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Command language

- x := E
  - x := x + 1
  - x := 10
- havoc x
- S ; T
- assert P
- assume P
- S □ T

Reasoning about execution traces

- Hoare triple { P } S { Q } says that

  every terminating execution trace of S that starts in a state satisfying P

  - does not go wrong, and
  - terminates in a state satisfying Q

Reasoning about execution traces

- Hoare triple { P } S { Q } says that

  every terminating execution trace of S that starts in a state satisfying P

  - does not go wrong, and
  - terminates in a state satisfying Q
- Given S and Q, what is the weakest P' satisfying {P'} S {Q} ?
  - P' is called the *weakest precondition* of S with respect to Q, written wp(S, Q)
  - to check {P} S {Q}, check $P \Rightarrow P'$

Weakest preconditions

- wp(x := E, Q) = Q[E / x]
- wp(havoc x, Q) = $(\forall x \bullet\ Q)$
- wp(assert P, Q) = $P \wedge Q$
- wp(assume P, Q) = $P \Rightarrow Q$
- wp(S ; T, Q) = wp(S, wp(T, Q))
- wp(S $\square$ T, Q) = wp(S, Q) $\wedge$ wp(T, Q)

Structured if statement

if E then S else T end =

assume E; S

□

assume ¬E; T

While loop with loop invariant

```
while E
      invariant J
do
      S
end

=     assert J;
      havoc x;  assume J;
      (    assume E;  S;  assert J;  assume false
      □    assume ¬E
      )
```

where x denotes the assignment targets of S

check that the loop invariant holds initially

"fast forward" to an arbitrary iteration of the loop

check that the loop invariant is maintained by the loop body

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Verification conditions: Structure

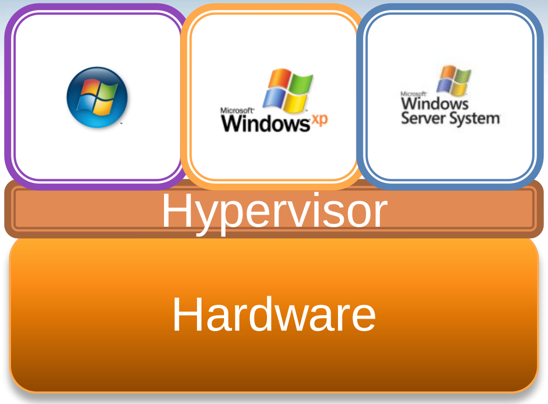
∀ Axioms (non-ground)

+

BIG and-or tree (ground)

Control & Data Flow

Hypervisor: A Manhattan Project



- **Meta OS**: small layer of software between hardware and OS
- **Mini**: 60K lines of non-trivial concurrent systems C code
- **Critical:** must provide functional resource abstraction
- **Trusted**: a verification grand challenge

Hypervisor: Some Statistics

- VCs have several Mb
- Thousands of non ground clauses
- Developers are willing to wait at most 5 min per VC

Challenge: annotation burden

- Partial solutions
  - Automatic generation of: Loop Invariants
  - Houdini-style automatic annotation generation

Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime

$$\forall\ h,o,f: \\ \quad IsHeap(h) \wedge o \neq null \wedge read(h, o, alloc) = t \\ \quad \Rightarrow \\ \quad read(h,o, f) = null \vee read(h, read(h,o,f),alloc) = t$$

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Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime
- Frame axioms

$$
\begin{aligned}
&\forall\ o, f: \\
&\qquad o \neq null \wedge read(h_0, o, alloc) = t \Rightarrow \\
&\qquad\quad read(h_1,o,f) = read(h_0,o,f) \vee (o,f) \in M
\end{aligned}
$$

Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime
- Frame axioms
- User provided assertions

  $\forall$ i,j: i $\leq$ j $\Rightarrow$ read(a,i) $\leq$ read(b,j)

Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime
- Frame axioms
- User provided assertions
- Theories

$\forall$ x: p(x,x)

$\forall$ x,y,z: p(x,y), p(y,z) $\Rightarrow$ p(x,z)

$\forall$ x,y: p(x,y), p(y,x) $\Rightarrow$ x = y

Challenge

- Quantifiers, quantifiers, quantifiers, …
- Modeling the runtime
- Frame axioms
- User provided assertions
- Theories
- Solver must be fast in satisfiable instances.

We want to find bugs!

Bad news

There is no sound and refutationally complete procedure for
linear integer arithmetic + free function symbols

Many Approaches

- Heuristic quantifier instantiation
- Combining SMT with Saturation provers
- Complete quantifier instantiation
- Decidable fragments
- Model based quantifier instantiation

Challenge: modeling runtime

- Is the axiomatization of the runtime consistent?
- False implies everything
- Partial solution: SMT + Saturation Provers
- Found many bugs using this approach

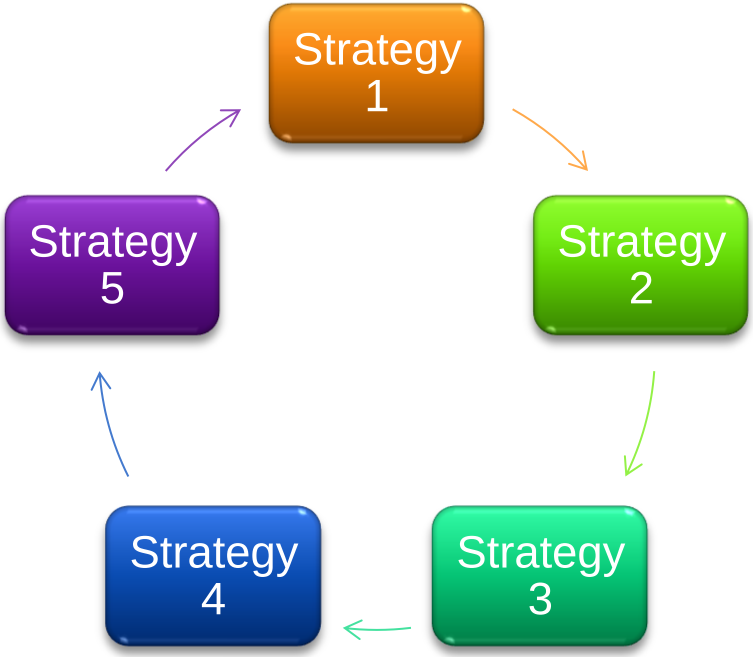
Challenge: Robustness

- Standard complain

  “I made a small modification in my Spec, and Z3 is timingout”
- This also happens with SAT solvers (NP-complete)
- In our case, the problems are undecidable
- Partial solution: parallelization

Parallel Z3

- Joint work with Y. Hamadi (MSRC) and C. Wintersteiger
- Multi-core & Multi-node (HPC)
- Different strategies in parallel
- Collaborate exchanging lemmas



Conclusion

- Logic as a platform
- Most verification/analysis tools need symbolic reasoning
- SMT is a hot area
- Many applications & challenges
- http://research.microsoft.com/projects/z3

Thank You!

E-matching & Quantifier instantiation

- SMT solvers use heuristic quantifier instantiation.
- E-matching (matching modulo equalities).
- Example:

$\forall\ x: f(g(x)) = x\ \{\ f(g(x))\ \}$

$a = g(b),$

$b = c,$

$f(a) \neq c$

Trigger

E-matching & Quantifier instantiation

- SMT solvers use heuristic quantifier instantiation.
- E-matching (matching modulo equalities).
- Example:

$\forall$ x: f(g(x)) = x { f(g(x)) }

a = g(b),

b = c,

f(a) $\neq$ c

x=b

f(g(b)) = b

Equalities and ground terms come from the partial model M

E-matching: why do we use it?

- Integrates smoothly with DPLL.
- Efficient for most VCs
- Decides useful theories:
  - Arrays
  - Partial orders
  - …

Efficient E-matching

- E-matching is NP-Hard.
- In practice

Problem	Indexing Technique
Fast retrieval	E-matching code trees
Incremental E-Matching	Inverted path index

E-matching code trees

Trigger:

f(x1, g(x1, a), h(x2), b)

Compiler

Instructions:

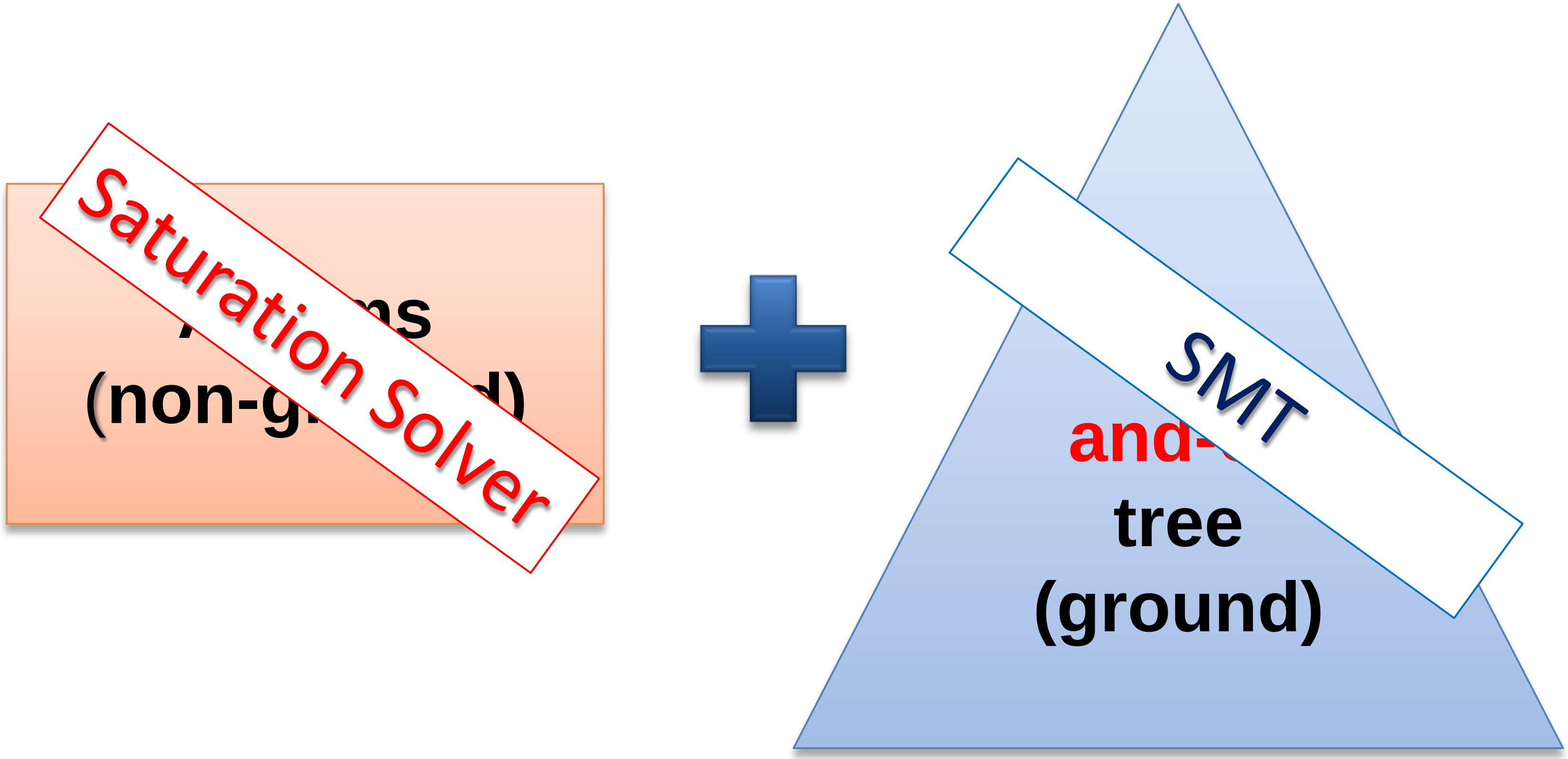
1. init(f, 2)
2. check(r4, b, 3)
3. bind(r2, g, r5, 4)
4. compare(r1, r5, 5)
5. check(r6, a, 6)
6. bind(r3, h, r7, 7)
7. yield(r1, r7)

Similar triggers share several instructions.

Combine code sequences in a code tree

DPLL(Γ)

- Tight integration: DPLL + Saturation solver.



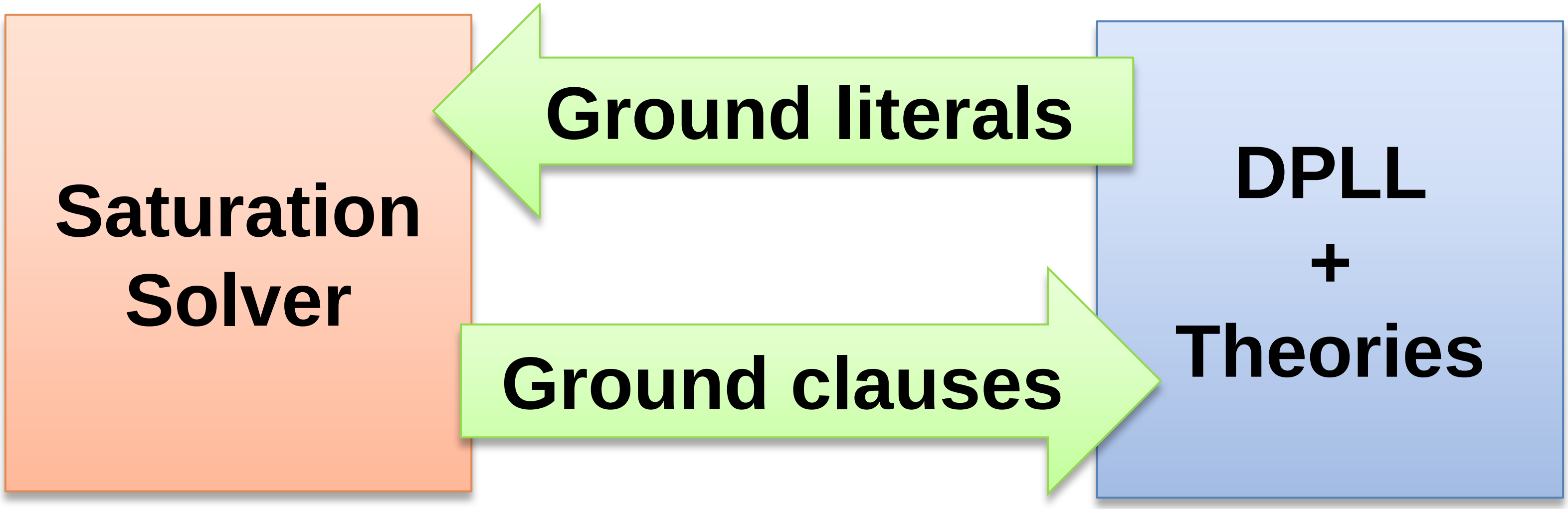
DPLL(Γ)

- Inference rule:

$$\frac{C_1 \quad \ldots \quad C_n}{C}$$

- DPLL(Γ) is parametric.
- Examples:
  - Resolution
  - Superposition calculus
  - …

DPLL(Γ)


Saturation Solver
Ground literals
Ground clauses
DPLL + Theories