Model-Driven Decision Procedures for Arithmetic

SYNASC 2013

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Microsoft Research

Logic Engines as a Service

FORMULA Modeling Foundations.

BOOGIE

VeriFast

Scala^{Z3}

SAGE

ESBMC

Isabelle

Pex

Vcc

SLAM

SecGuru

Yogi

TERMINATOR

Spec Explorer

The Spec# Programming System

Satisfiability

$x^2 + y^2 < 1 \; and \; xy > 0.1$ → sat, $x = \frac{1}{8}, y = \frac{7}{8}$ (Solution/Model)

$x^2 + y^2 < 1 \; and \; xy > 1$ → unsat, Proof

Is execution path *P* feasible?

SAGE

Is assertion *X* violated?

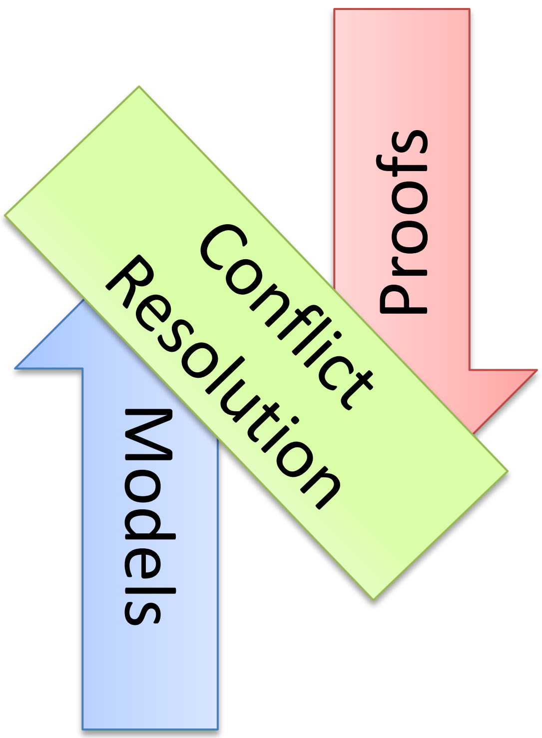
WITNESS

Is Formula *F* Satisfiable?

The RISE of Model-Driven Techniques

Saturation x Search

Proof-finding Model-finding



SAT

$$p_1 \lor \neg p_2, \qquad \neg p_1 \lor p_2 \lor p_3, \qquad p_3$$

$$p_1 = true, \qquad p_2 = true, \qquad p_3 = true$$

CNF is a set (conjunction) set of clauses
Clause is a disjunction of literals
Literal is an atom or the negation of an atom

Two procedures

Resolution	DPLL
Proof-finder	Model-finder
Saturation	Search

Resolution

$$C \vee l, \quad D \vee \neg l \quad \Rightarrow \quad C \vee D$$

$$l, \neg l \quad \Rightarrow \quad \textbf{unsat}$$

Improvements

Delete tautologies $l \vee \neg l \vee C$

Ordered Resolution

Subsumption (delete redundant clauses)

C *subsumes* $C \vee D$

…

Resolution: Example

$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r$

Resolution: Example

$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r \quad \Rightarrow$

$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r,\ \neg q \vee r$

Resolution: Example

$$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r \qquad \Rightarrow$$

$$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r,\ \neg q \vee r \qquad \Rightarrow$$

$$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r,\ \neg q \vee r,\ q \vee r$$

Resolution: Example

$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r \qquad \Rightarrow$

$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r,\ \neg q \vee r \qquad \Rightarrow$

$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r,\ {\color{red}\neg q} \vee r,\ {\color{red}q} \vee r \qquad \Rightarrow$

$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r,\ \neg q \vee r,\ q \vee r,\ {\color{red}r}$

Resolution: Example

$$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r \quad \Rightarrow$$

$$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r,\ \neg q \vee r \quad \Rightarrow$$

$$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r,\ \neg q \vee r,\ q \vee r \quad \Rightarrow$$

$$\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p \vee r,\ \neg r,\ \neg q \vee r,\ q \vee r,\ r \quad \Rightarrow$$

unsat

Resolution: Problem

Exponential time and space

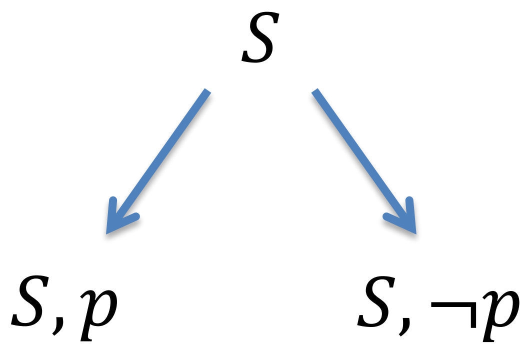
Unit Resolution

$$C \vee l, \ \neg l \quad \Rightarrow \quad C$$

C
subsumes
$C \vee l$

DPLL

Split rule



DPLL = Unit Resolution + Split rule

DPLL

$x \vee y, \quad \neg x \vee y, \quad x \vee \neg y, \quad \neg x \vee \neg y$

$x \vee y,$
$\neg x \vee y,$
$x \vee \neg y,$
$\neg x \vee \neg y,$
x

DPLL

$x \vee y, \quad \neg x \vee y, \quad x \vee \neg y, \quad \neg x \vee \neg y$

$x \vee y,$

$\neg x \vee y,$

$x \vee \neg y,$

$\neg x \vee \neg y,$

x

DPLL

$x \lor y$, $\neg x \lor y$, $x \lor \neg y$, $\neg x \lor \neg y$

y,

$\neg y$,

x

DPLL

$x \lor y, \quad \neg x \lor y, \quad x \lor \neg y, \quad \neg x \lor \neg y$

$y,$

$\neg y,$

$x,$

$unsat$

DPLL

$x \lor y, \quad \neg x \lor y, \quad x \lor \neg y, \quad \neg x \lor \neg y$

$y,$
$\neg y,$
$x,$
$unsat$

$x \lor y,$
$\neg x \lor y,$
$x \lor \neg y,$
$\neg x \lor \neg y,$
$\neg x$

DPLL

$x \vee y, \quad \neg x \vee y, \quad x \vee \neg y, \quad \neg x \vee \neg y$

$y,$

$\neg y,$

$x,$

$unsat$

$x \vee y,$

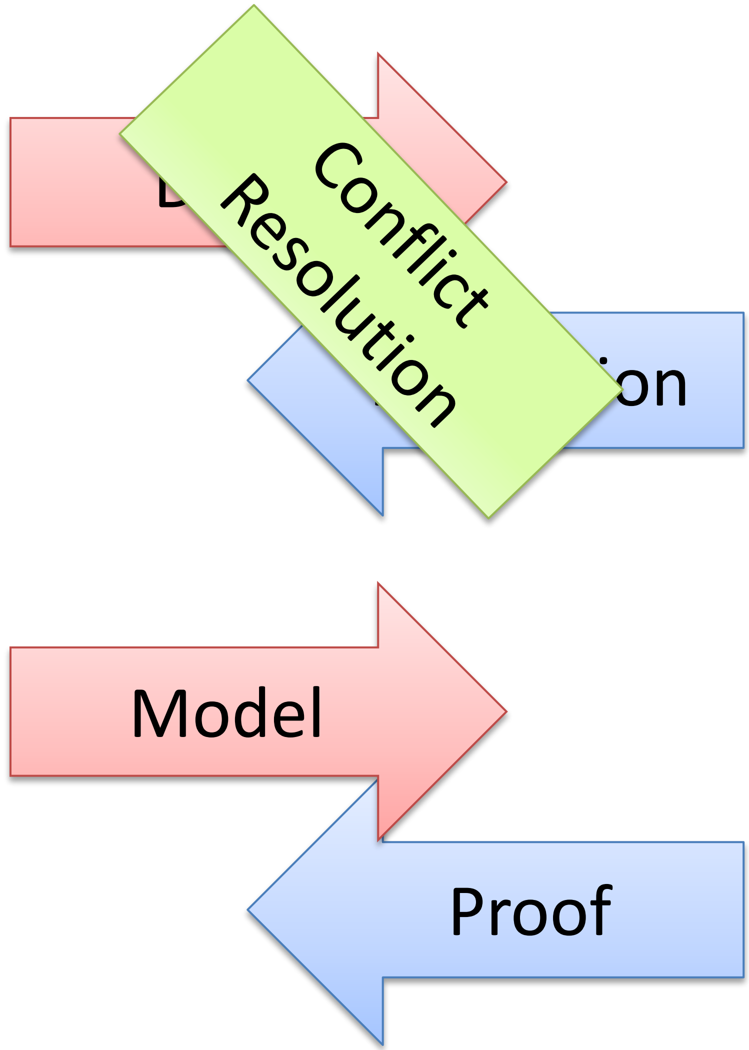
$\neg x \vee y,$

$x \vee \neg y,$

$\neg x \vee \neg y,$

$\neg x$

CDCL: Conflict Driven Clause Learning



Linear Arithmetic

Fourier-Motzkin	Simplex
Proof-finder	Model-finder
Saturation	Search

Fourier-Motzkin

$$t_1 \leq ax, \qquad bx \leq t_2$$

$$\Downarrow$$

$$bt_1 \leq abx, \qquad abx \leq at_2$$

$$\Downarrow$$

$$bt_1 \leq at_2$$

Very similar to Resolution

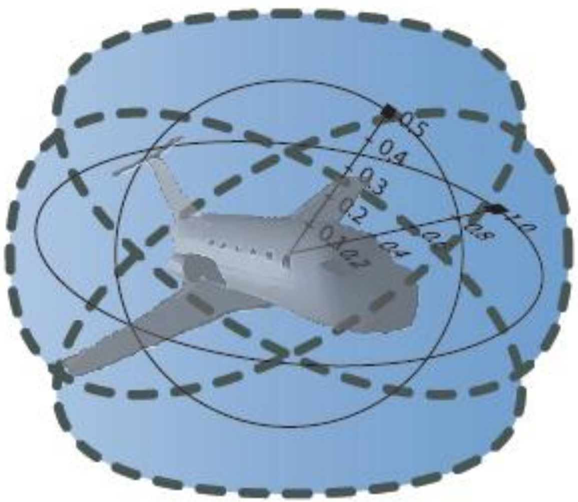
Exponential time and space

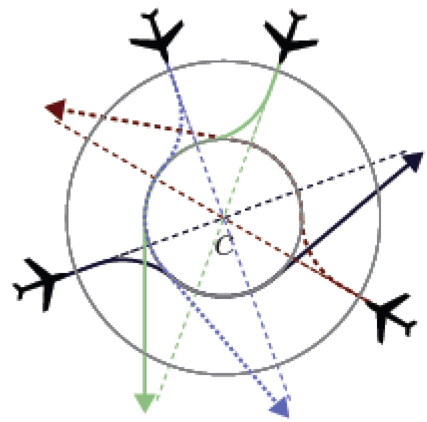
Polynomial Constraints

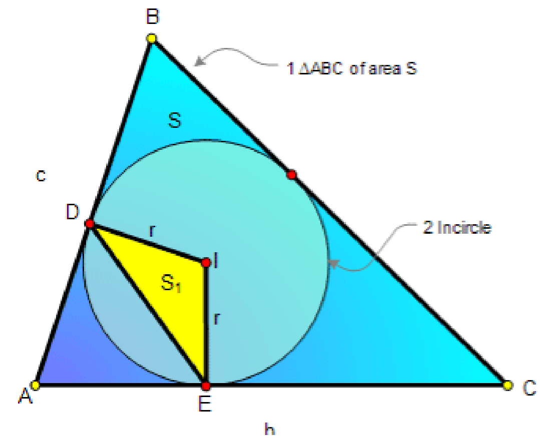
AKA
Existential Theory of the Reals
∃R

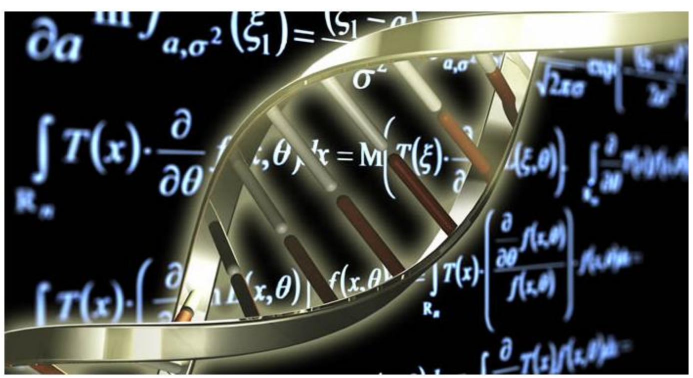
$$x^2 - 4x + y^2 - y + 8 < 1$$
$$xy - 2x - 2y + 4 > 1$$

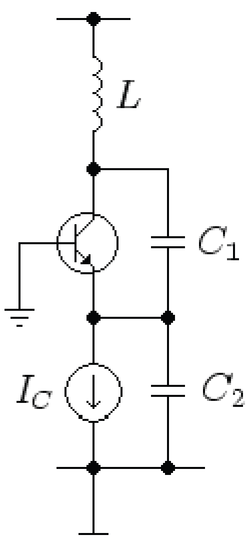
Applications











CiME

CAD "Big Picture"

1. Project/Saturate set of polynomials
2. Lift/Search: Incrementally build assignment v: $x_k \rightarrow \alpha_k$

   Isolate roots of polynomials $f_i(\boldsymbol{\alpha}, x)$

   Select a feasible cell C, and assign x_k some $\alpha_k \in C$

   If there is no feasible cell, then backtrack

CAD "Big Picture"

$x^2 + y^2 - 1 < 0$
$x\,y - 1 > 0$

→ 1. Saturate

$x^4 - x^2 + 1$
$x^2 - 1$
x

2. Search

	$(-\infty, -1)$	-1	$(-1, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$x^4 - x^2 + 1$	+	+	+	+	+	+	+
$x^2 - 1$	+	0	-	-	-	0	+
x	-	-	-	0	+	+	+

CAD "Big Picture"

$x^2 + y^2 - 1 < 0$

$x\,y - 1 > 0$

1. Saturate

$x^4 - x^2 + 1$

$x^2 - 1$

x

	$(-\infty, -\frac{1}{2})$	$-\frac{1}{2}$	$(-\frac{1}{2}, \infty)$
$4 + y^2 - 1$	+	+	+
$-2y - 1$	+	0	-

$x \to -2$

2. Search

	$(-\infty, -1)$	-1	$(-1, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$x^4 - x^2 + 1$	+	+	+	+	+	+	+
$x^2 - 1$	+	0	-	-	-	0	+
x	-	-	-	0	+	+	+

CAD "Big Picture"

$\boldsymbol{x^2 + y^2 - 1 < 0}$

$x\,y - 1 > 0$

1. Saturate

$x^4 - x^2 + 1$

$x^2 - 1$

x

	$(-\infty, -\frac{1}{2})$	$-\frac{1}{2}$	$(-\frac{1}{2}, \infty)$
$\boldsymbol{4 + y^2 - 1}$	**+**	**+**	**+**
$-2y - 1$	+	0	-

CONFLICT

$\boldsymbol{x \to -2}$

2. Search

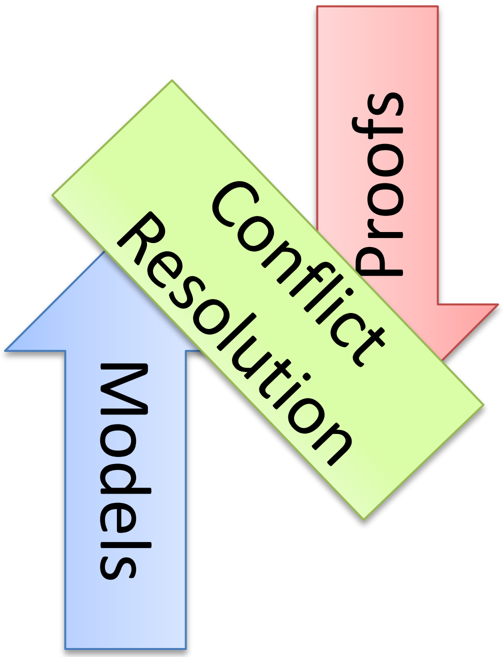
	$(-\infty, -1)$	-1	$(-1, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$x^4 - x^2 + 1$	+	+	+	+	+	+	+
$x^2 - 1$	+	0	-	-	-	0	+
x	-	-	-	0	+	+	+

NLSAT: Model-Based Search

Static x Dynamic

Optimistic approach

Key ideas



Start the Search before Saturate/Project

We saturate on demand

Model guides the saturation

NLSAT (1)

Two kinds of decision

1. case-analysis (Boolean)

$$x^2 + y^2 < 1 \lor \boldsymbol{x < 0} \lor x\,y > 1$$

2. model construction (CAD lifting)

$$\boldsymbol{x \rightarrow -2}$$

	$(-\infty, -1)$	-1	$(-1, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$x^4 - x^2 + 1$	+	+	+	+	+	+	+
$x^2 - 1$	+	0	-	-	-	0	+
x	-	-	-	0	+	+	+

NLSAT (1)

Two kinds of decision

1. case-analysis (Boolean)
2. model construction (CAD lifting)

Parametric calculus: $explain(F, M)$

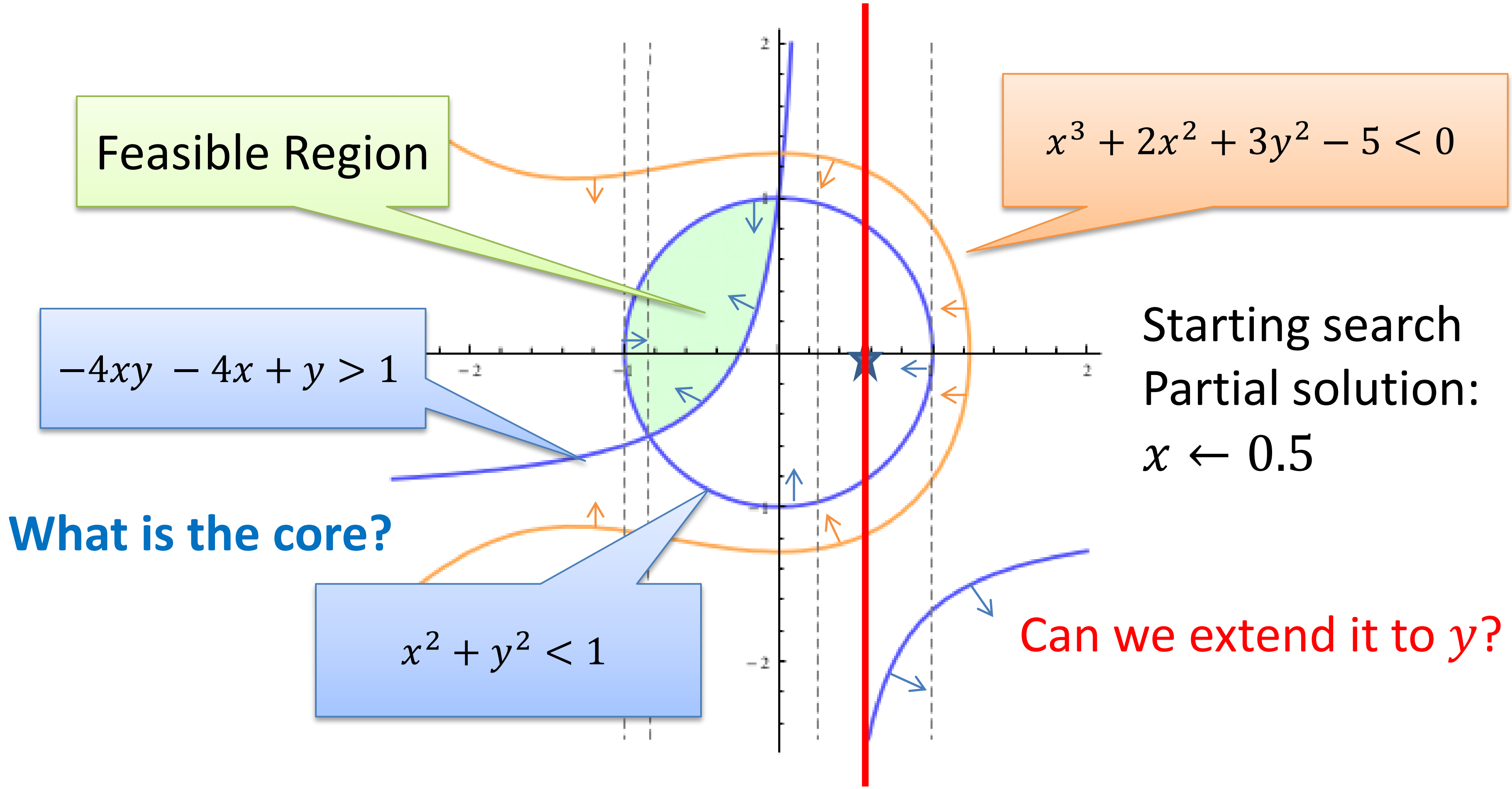
Finite basis explanation function

Explanations may contain new literals

They evaluate to false in the current state

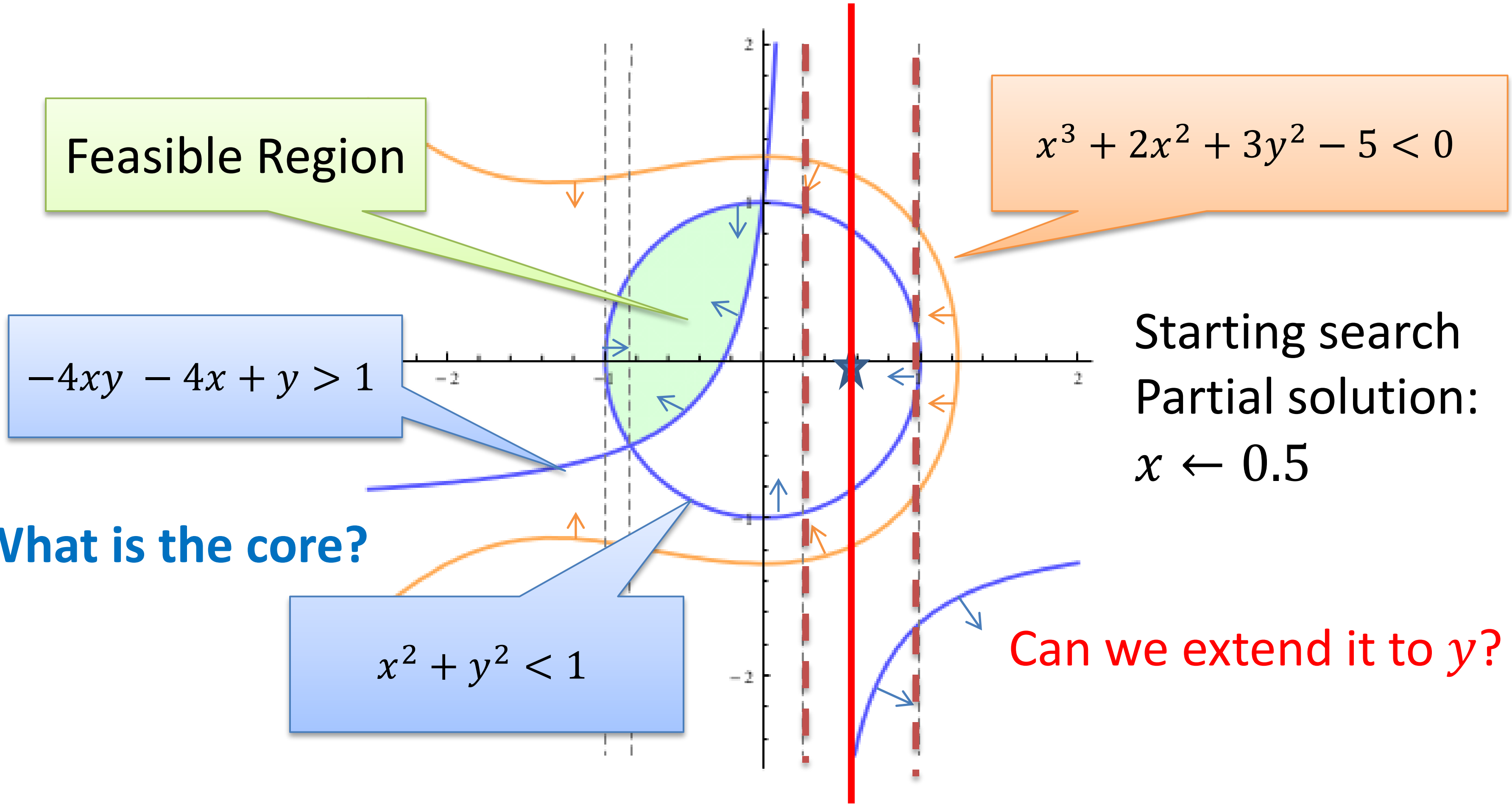
NLSAT (2)

Key ideas: Use partial solution to guide the search



NLSAT (2)

Key ideas: Use partial solution to guide the search



Starting search
Partial solution:
$x \leftarrow 0.5$

What is the core?

Can we extend it to y?

NLSAT (3)

Key ideas: Solution based Project/Saturate

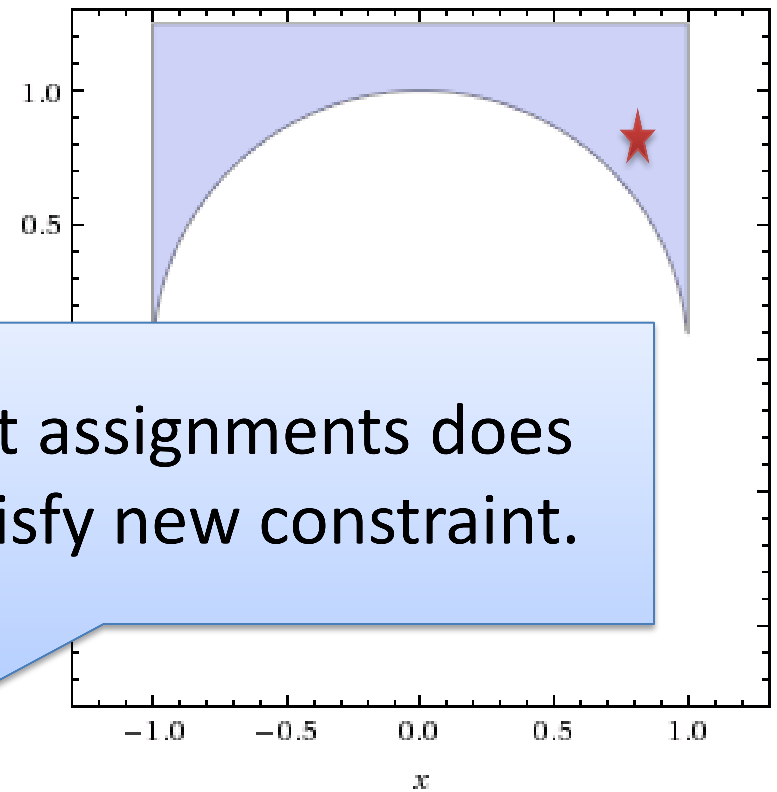
$$\mathsf{P}_c(A, x) = \bigcup_{f \in A} \mathsf{coeff}(f, x) \cup \bigcup_{\substack{f \in A \\ g \in \mathsf{R}(f,x)}} \mathsf{psc}(g, g'_x, x) \cup \bigcup_{\substack{i<j \\ g_i \in \mathsf{R}(f_i,x) \\ g_j \in \mathsf{R}(f_j,x)}} \mathsf{psc}(g_i, g_j, x)$$

Standard project operators are pessimistic.
Coefficients can vanish!

NLSAT (4)

Key ideas: Lemma Learning

Prevent a **Conflict** from happening again.



Current assignment

$x \to 0.75$

$y \to 0.75$

Conflict

$x^2 + y^2 + z^2 < 1$

Current assignments does not satisfy new constraint.

Lemma

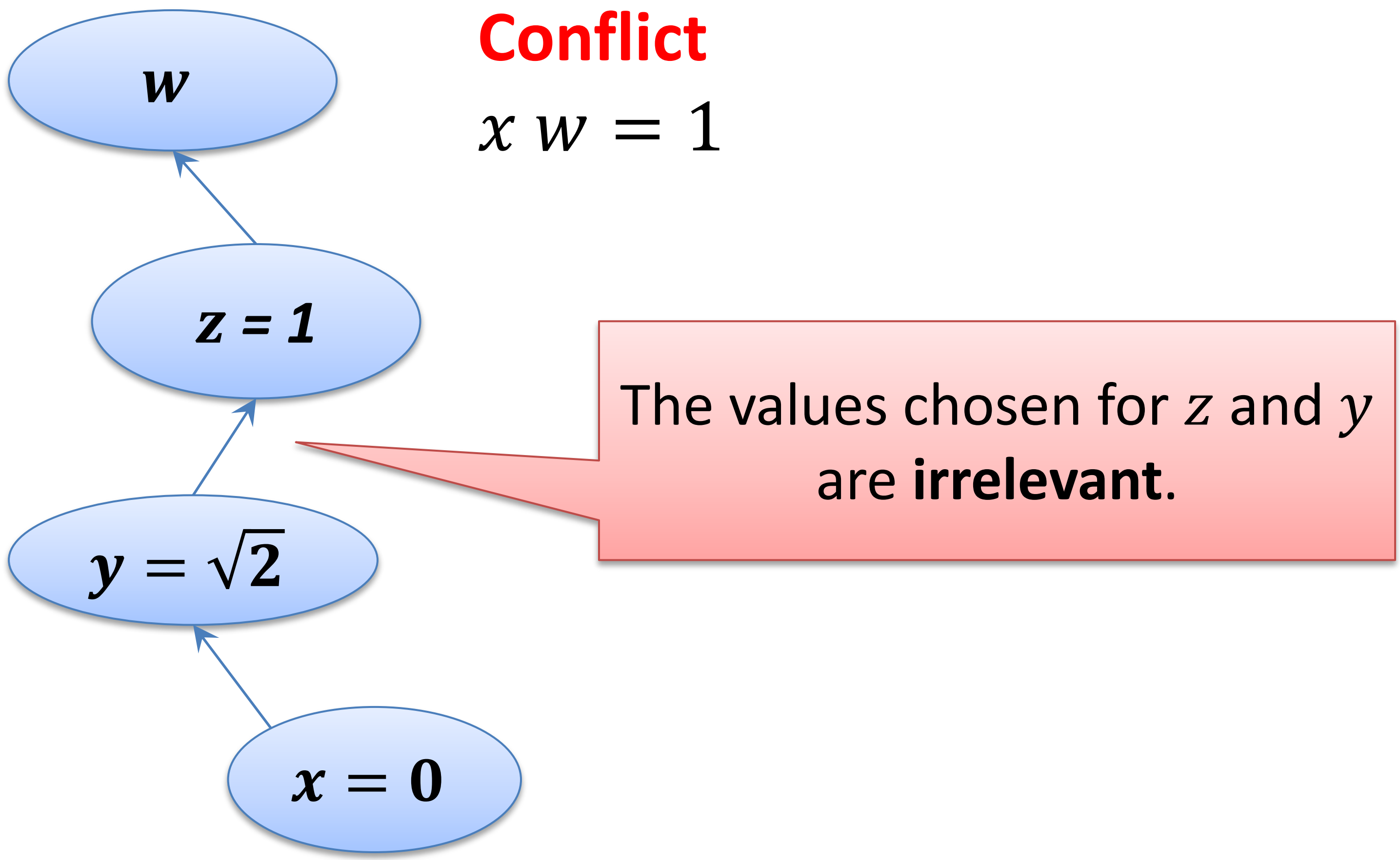
$$-1 < x < 1 \ \wedge \ y > root_2(1 - \tilde{y}^2 - x^2) \ \Rightarrow \bot$$

NLSAT (5)

Key ideas: Nonchronological Backtracking

Conflict

$x\,w = 1$



Machinery

Multivariate & univariate Polynomials

Basic operations, Pseudo-division,

GCD, Resultant, PSC, Factorization,

Root isolation algorithms, Sturm sequences

Binary rationals $\frac{a}{2^k}$

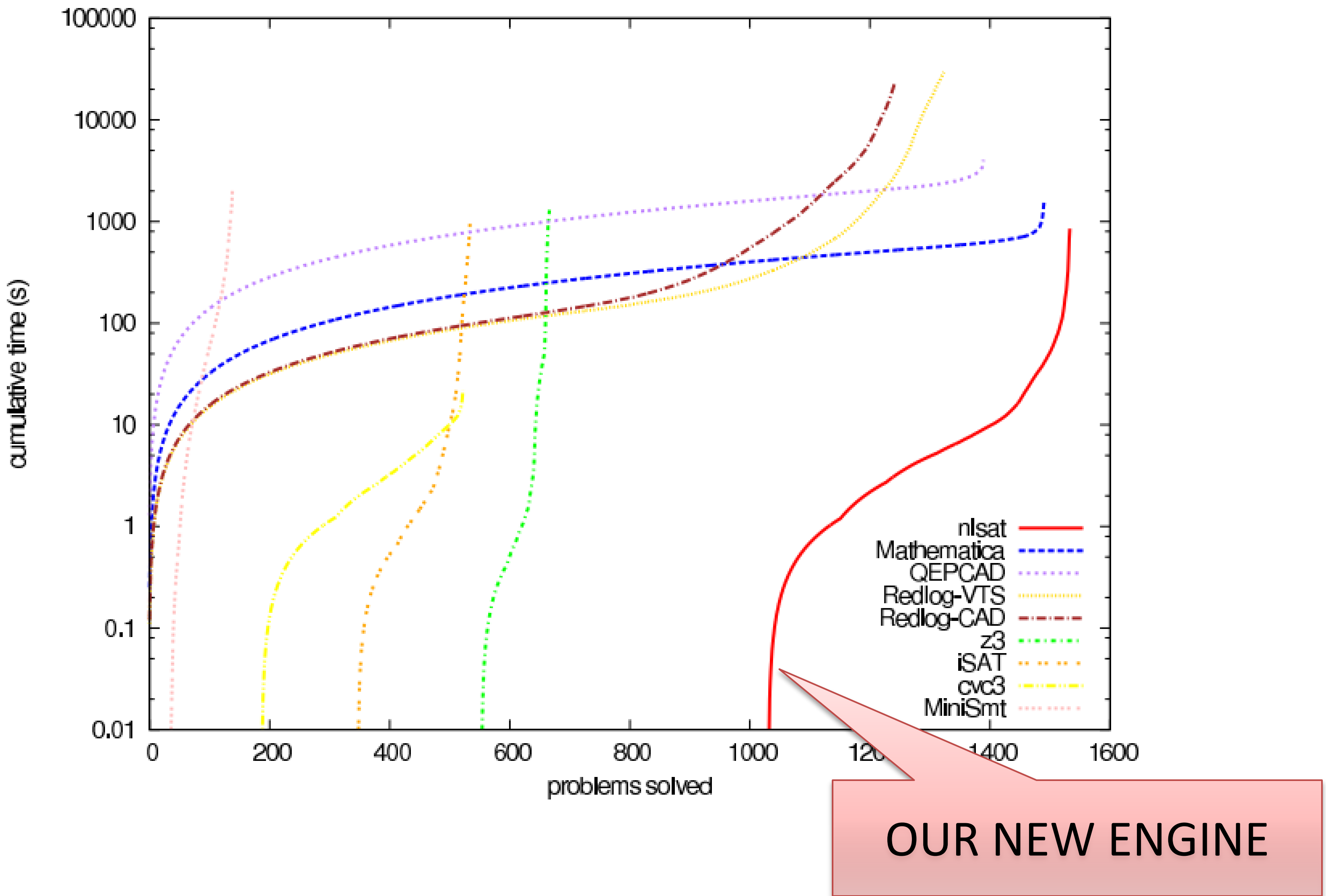
Real Algebraic Numbers

Experimental Results (1)

OUR NEW ENGINE

solver	meti-tarski (1006) solved	meti-tarski (1006) time (s)	keymaera (421) solved	keymaera (421) time (s)	zankl (166) solved	zankl (166) time (s)	hong (20) solved	hong (20) time (s)	kissing (45) solved	kissing (45) time (s)	all (1658) solved	all (1658) time (s)
nlsat	1002	343	**420**	**5**	**89**	**234**	10	170	13	95	**1534**	**849**
Mathematica	**1006**	**796**	420	171	50	366	9	208	6	29	1491	1572
QEPCAD	991	2616	368	1331	21	38	6	43	4	5	1390	4036
Redlog-VTS	847	28640	419	78	42	490	6	3	10	275	1324	29488
Redlog-CAD	848	21706	363	730	21	173	6	2	4	0	1242	22613
z3	266	83	379	1216	21	0	1	0	0	0	667	1299
iSAT	203	122	291	16	21	24	**20**	**822**	0	0	535	986
cvc3	150	13	361	5	12	3	0	0	0	0	523	22
MiniSmt	40	697	35	0	46	1370	0	0	**18**	**44**	139	2112

Experimental Results (2)



Other examples
(for linear arithmetic)

Fourier-Motzkin **X** Generalizing DPLL to richer logics

[McMillan et al 2009]

Conflict Resolution

[Korovin et al 2009]

Other examples

Array Theory by
Axiom Instantiation

X

Lemmas on Demand
For Theory of Array
[Brummayer-Biere 2009]

$$\forall a, i, v: \quad a[i := v][i] = v$$
$$\forall a, i, j, v: \ i = j \lor a[i := v][j] = a[j]$$

Saturation: successful instances

Polynomial time procedures

Gaussian Elimination

Congruence Closure

MCSat

Model-Driven SMT

Lift ideas from CDCL to SMT

Generalize ideas found in model-driven approaches

Easier to implement

Model construction is explicit

MCSat

$x \geq 2, \qquad (\neg x \geq 1 \lor y \geq 1), \qquad (x^2 + y^2 \leq 1 \lor xy > 1)$

MCSat

$x \geq 2, \qquad (\neg x \geq 1 \vee y \geq 1), \qquad (x^2 + y^2 \leq 1 \vee xy > 1)$

$x \geq 2$

Propagations

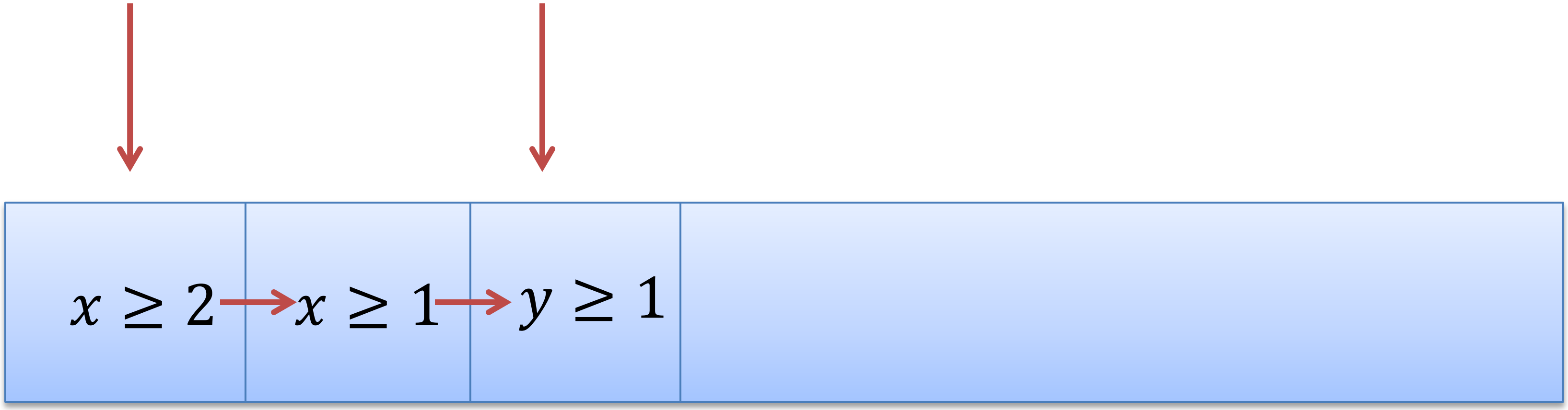
MCSat

$x \geq 2$, $(\neg x \geq 1 \vee y \geq 1)$, $(x^2 + y^2 \leq 1 \vee xy > 1)$

Propagations

MCSat

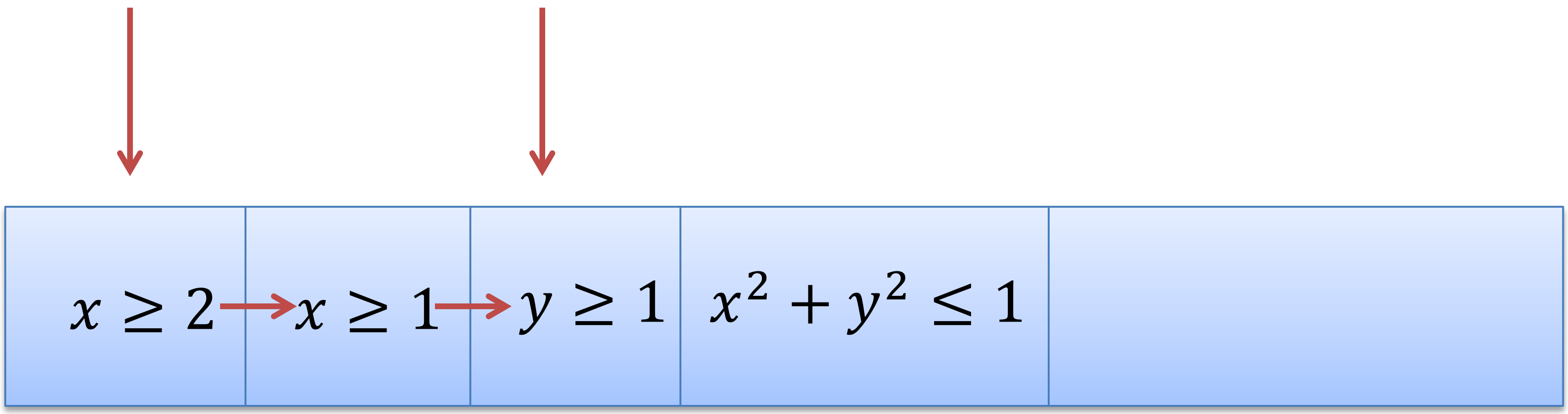
$x \geq 2, \quad (\neg x \geq 1 \vee y \geq 1), \quad (x^2 + y^2 \leq 1 \vee xy > 1)$



Propagations

MCSat

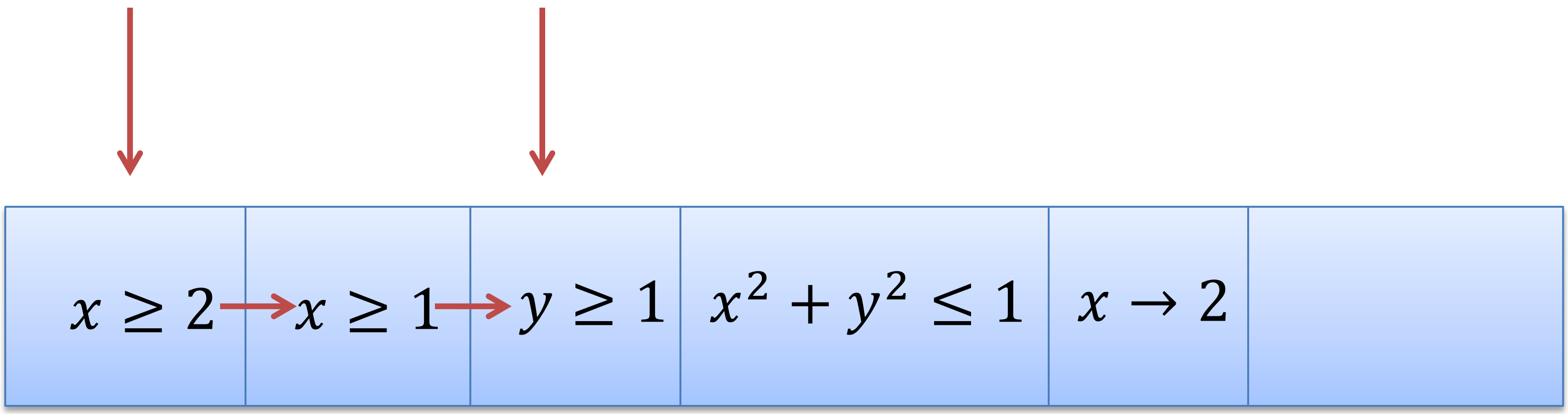
$x \geq 2, \quad (\neg x \geq 1 \vee y \geq 1), \quad (x^2 + y^2 \leq 1 \vee xy > 1)$



Boolean Decisions

MCSat

$$x \geq 2, \quad (\neg x \geq 1 \lor y \geq 1), \quad (x^2 + y^2 \leq 1 \lor xy > 1)$$



Semantic Decisions

MCSat

$x \geq 2, \quad (\neg x \geq 1 \lor y \geq 1), \quad (x^2 + y^2 \leq 1 \lor xy > 1)$

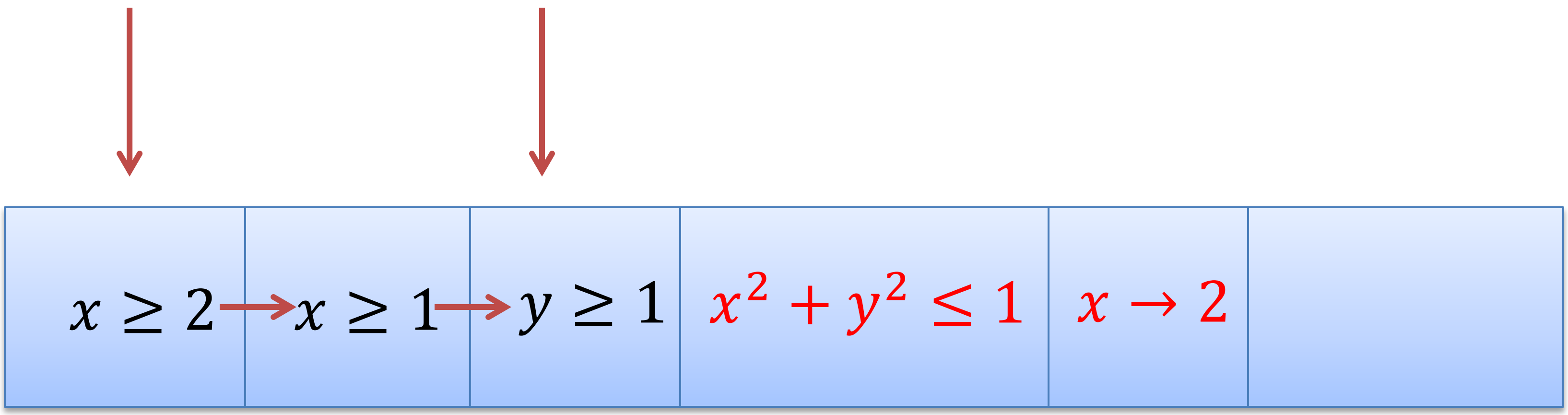
$x \geq 2$	$x \geq 1$	$y \geq 1$	$x^2 + y^2 \leq 1$	$x \to 2$	

Conflict

We can't find a value for y
s.t. $4 + y^2 \leq 1$

MCSat

$x \geq 2$, $(\neg x \geq 1 \lor y \geq 1)$, $(x^2 + y^2 \leq 1 \lor xy > 1)$



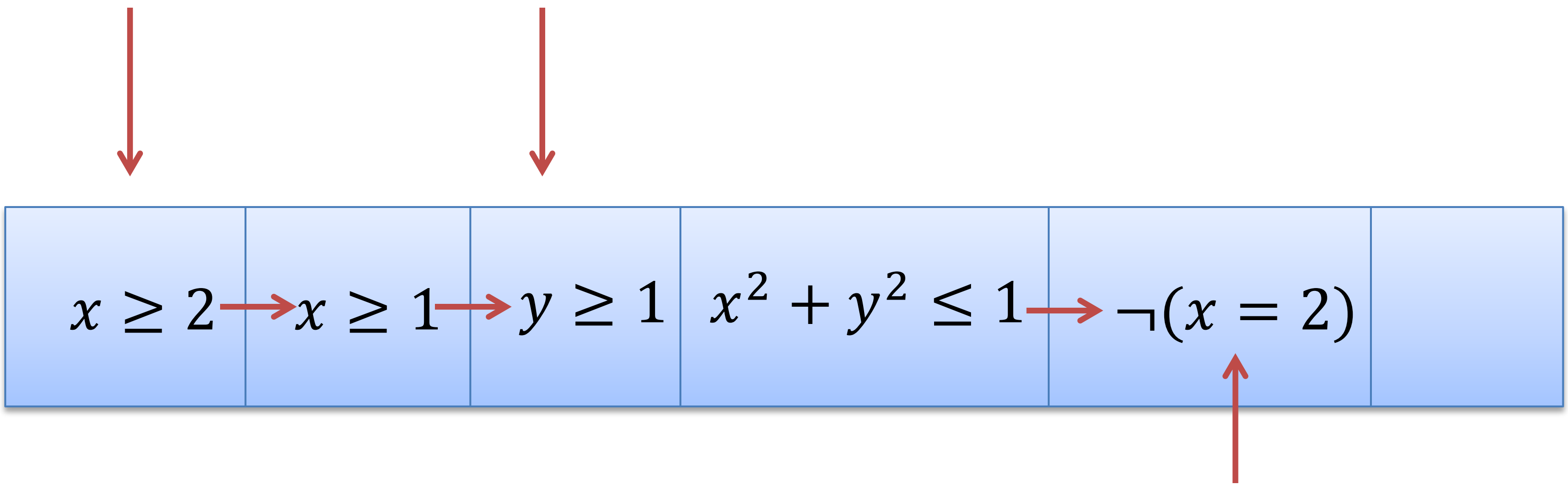
Conflict

We can't find a value for y s.t. $4 + y^2 \leq 1$

Learning that $\neg(x^2 + y^2 \leq 1) \lor \neg(x = 2)$ is not productive

MCSat

$x \geq 2$, $(\neg x \geq 1 \lor y \geq 1)$, $(x^2 + y^2 \leq 1 \lor xy > 1)$

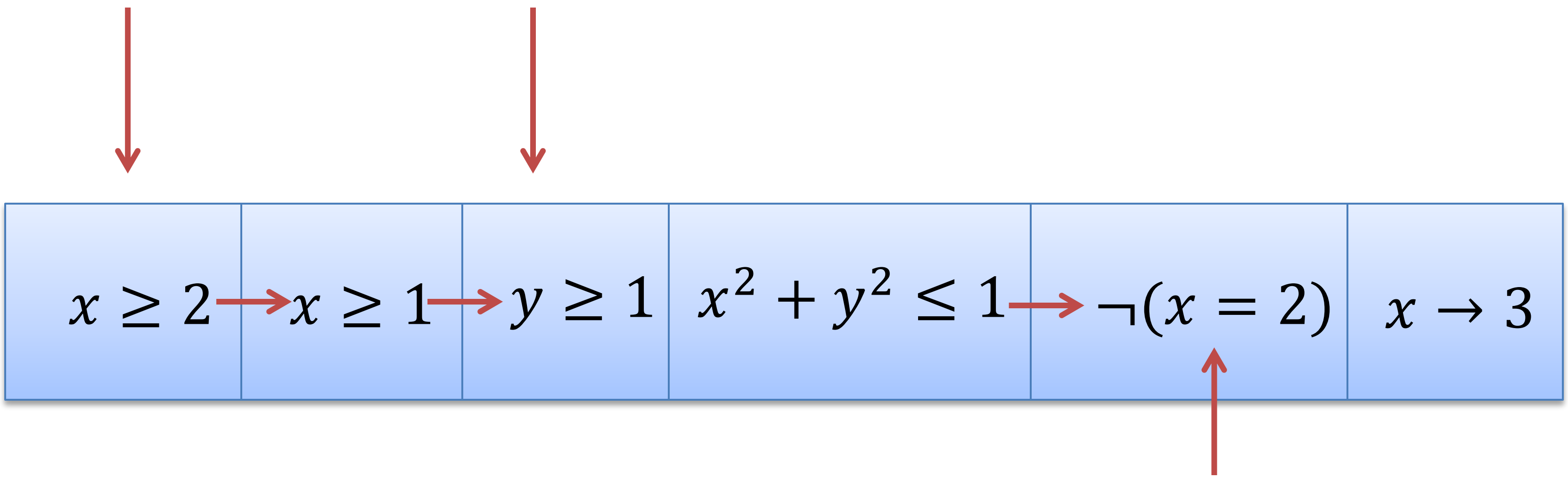


$\neg(x^2 + y^2 \leq 1) \lor \neg(x = 2)$

Learning that
$\neg(x^2 + y^2 \leq 1) \lor \neg(x = 2)$
is not productive

MCSat

$x \geq 2$, $(\neg x \geq 1 \vee y \geq 1)$, $(x^2 + y^2 \leq 1 \vee xy > 1)$



$\neg(x^2 + y^2 \leq 1) \vee \neg(x = 2)$

Learning that
$\neg(x^2 + y^2 \leq 1) \vee \neg(x = 2)$
is not productive

MCSat

$x \geq 2$, $(\neg x \geq 1 \lor y \geq 1)$, $(x^2 + y^2 \leq 1 \lor xy > 1)$

$x \geq 2$	$x \geq 1$	$y \geq 1$	$x^2 + y^2 \leq 1$	$\neg(x = 2)$	$x \to 3$

"Same" Conflict

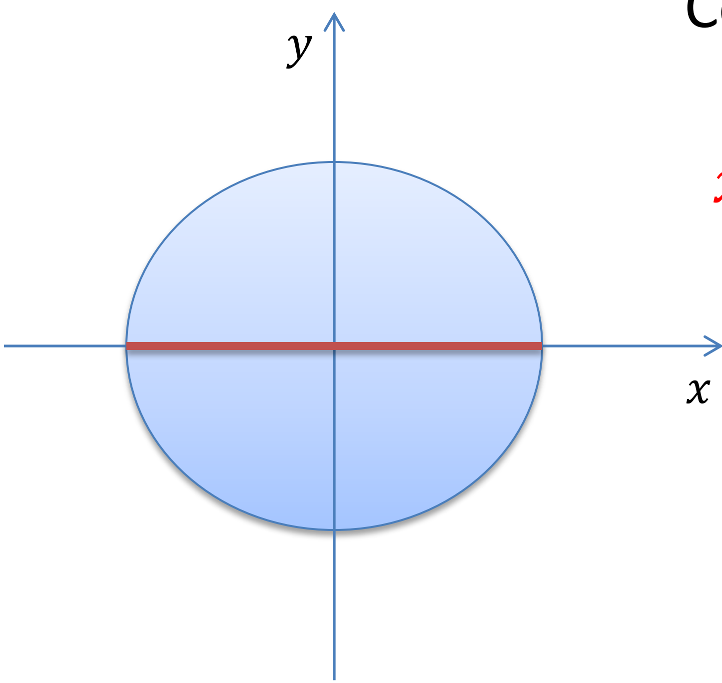
$\neg(x^2 + y^2 \leq 1) \lor \neg(x = 2)$

We can't find a value for y
s.t. $9 + y^2 \leq 1$

Learning that
$\neg(x^2 + y^2 \leq 1) \lor \neg(x = 2)$
is not productive

$x \geq 2, \quad (\neg x \geq 1 \lor y \geq 1), \quad (x^2 + y^2 \leq 1 \lor xy > 1)$

$x \geq 2$ →	$x \geq 1$ →	$y \geq 1$	$x^2 + y^2 \leq 1$	$x \to 2$	



Conflict

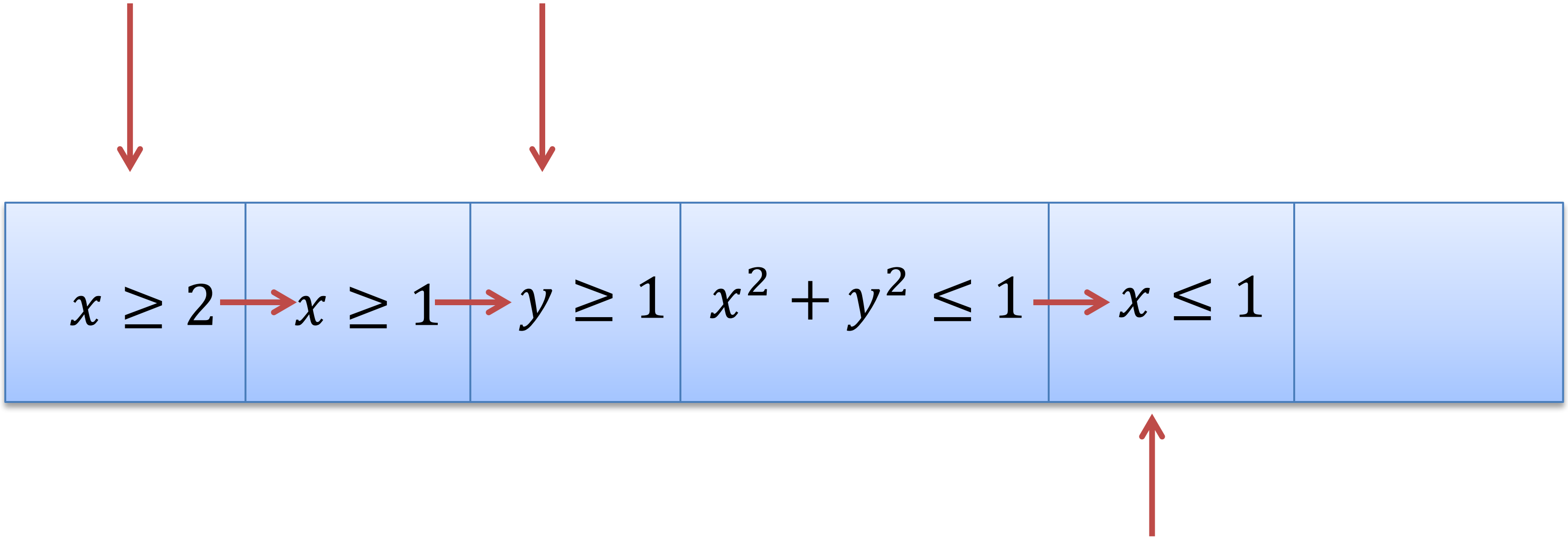
$x^2 + y^2 \leq 1$ → $-1 \leq x, x \leq 1$

$x \to 2$

$\neg(x^2 + y^2 \leq 1) \lor x \leq 1$

MCSat

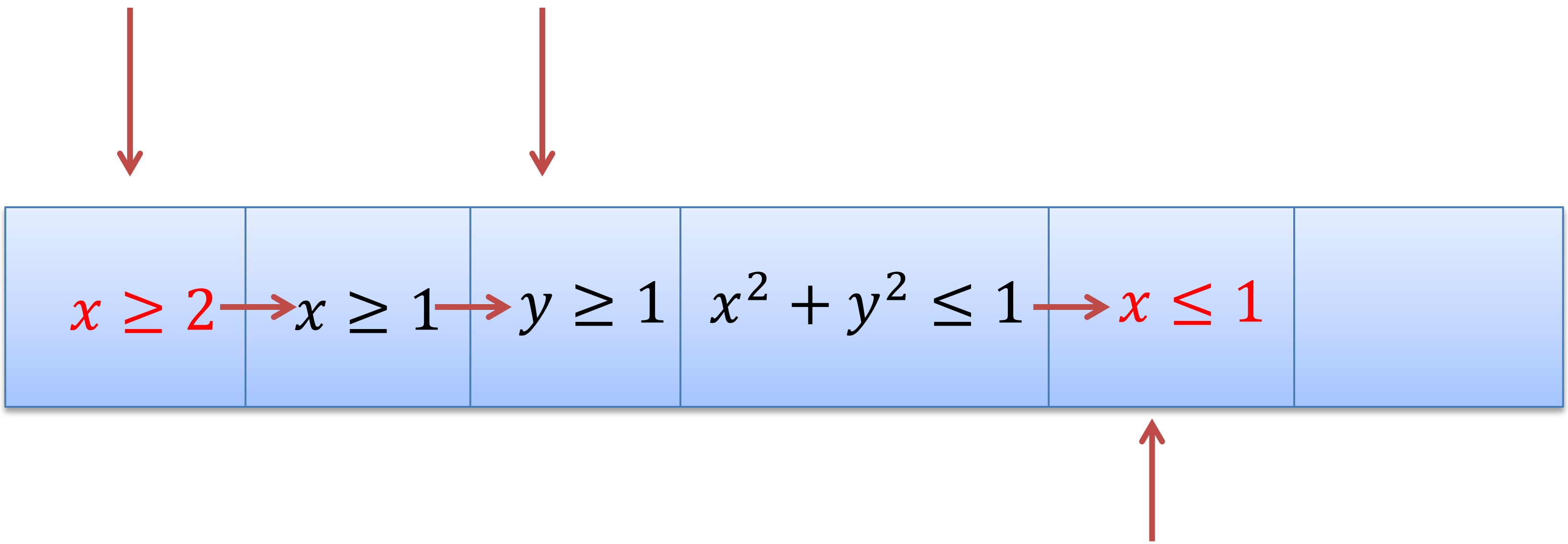
$x \geq 2, \quad (\neg x \geq 1 \vee y \geq 1), \quad (x^2 + y^2 \leq 1 \vee xy > 1)$



$\neg(x^2 + y^2 \leq 1) \vee x \leq 1$

MCSat

$x \geq 2, \quad (\neg x \geq 1 \vee y \geq 1), \quad (x^2 + y^2 \leq 1 \vee xy > 1)$



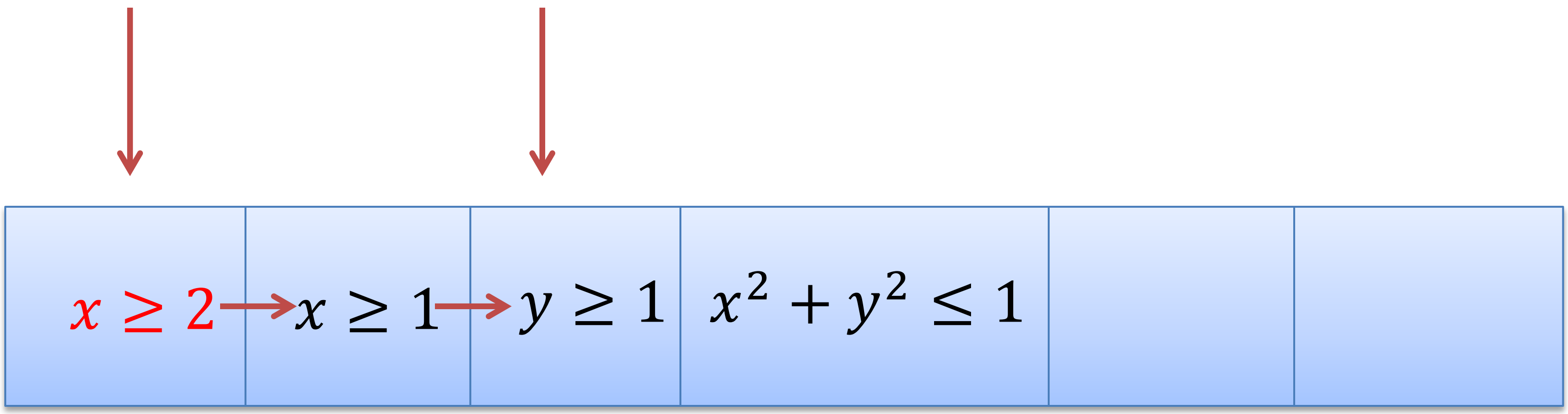
$\neg(x^2 + y^2 \leq 1) \vee x \leq 1$

Conflict

$\neg(x \geq 2) \vee \neg(x \leq 1)$

MCSat

$x \geq 2, \quad (\neg x \geq 1 \vee y \geq 1), \quad (x^2 + y^2 \leq 1 \vee xy > 1)$



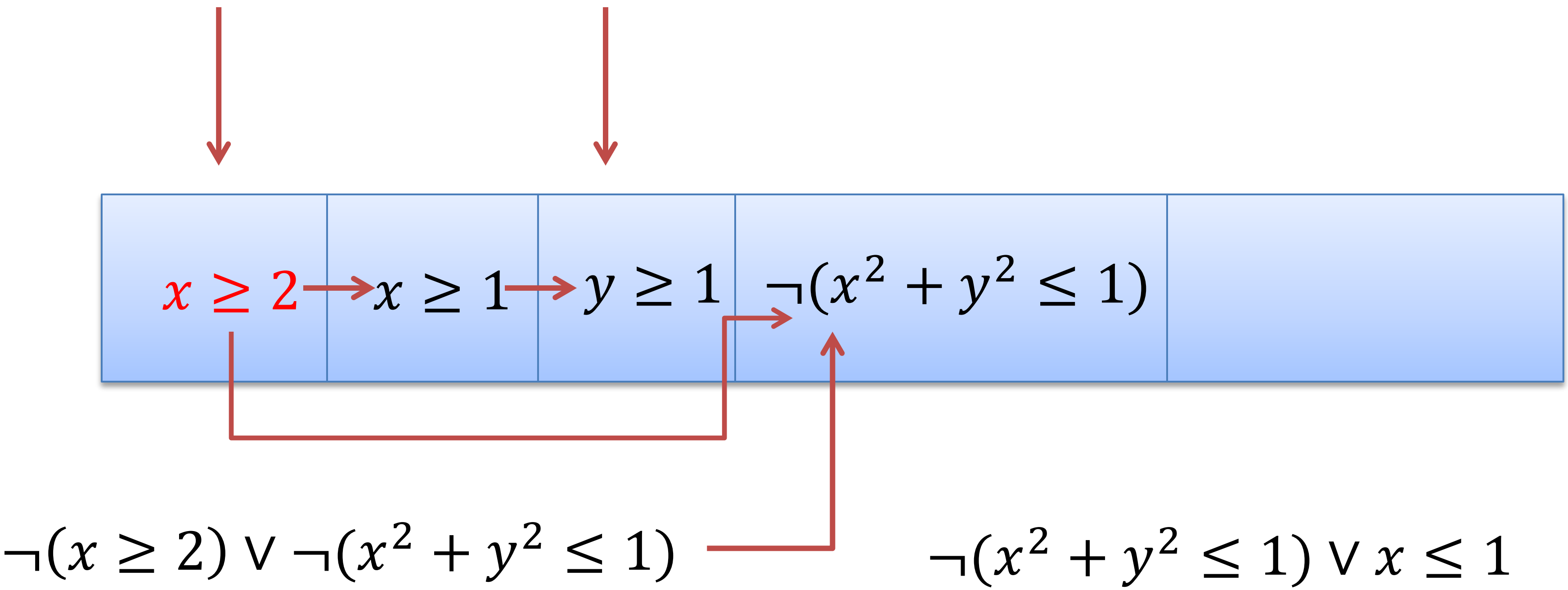
$\neg(x^2 + y^2 \leq 1) \vee x \leq 1$

Learned by resolution

$\neg(x \geq 2) \vee \neg(x^2 + y^2 \leq 1)$

MCSat

$x \geq 2$, $(\neg x \geq 1 \vee y \geq 1)$, $(x^2 + y^2 \leq 1 \vee xy > 1)$



MCSat: FM Example

$-x + z + 1 \leq 0$	$z \to 0$	$x - y \leq 0$	$y \to 0$	

$$-x + z + 1 \leq 0, \quad x - y \leq 0 \qquad\qquad z \to 0, \quad y \to 0$$

$$\equiv$$

$$z + 1 \leq x, \quad x \leq y$$

$$1 \leq x, \quad x \leq 0$$

We can't find a value of x

MCSat: FM Example

$-x+z+1 \leq 0$	$z \to 0$	$x-y \leq 0$	$y \to 0$	

$-x+z+1 \leq 0, \quad x-y \leq 0$ $\qquad z \to 0, \quad y \to 0$

$\exists x: -x+z+1 \leq 0 \ \wedge \ x-y \leq 0$

$z+1-y \leq 0$

Fourier-Motzkin

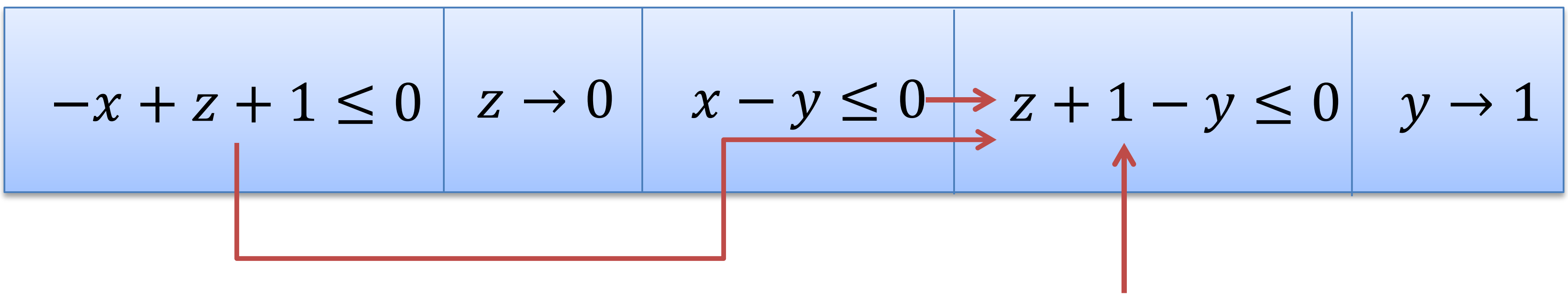
$\neg(-x+z+1 \leq 0) \vee \neg(x-y \leq 0) \vee z+1-y \leq 0$

MCSat: FM Example

$-x + z + 1 \leq 0$	$z \to 0$	$x - y \leq 0$	$z + 1 - y \leq 0$	

$$\neg(-x + z + 1 \leq 0) \vee \neg(x - y \leq 0) \vee z + 1 - y \leq 0$$

MCSat: FM Example



$$\neg(-x + z + 1 \leq 0) \vee \neg(x - y \leq 0) \vee z + 1 - y \leq 0$$

$$-x + z + 1 \leq 0, \quad x - y \leq 0 \qquad\qquad z \to 0, \quad y \to 1$$

$$\equiv$$

$$z + 1 \leq x, \quad x \leq y$$

$$1 \leq x, \quad x \leq 1$$

MCSat: FM Example

$-x + z + 1 \leq 0$	$z \to 0$	$x - y \leq 0$	$z + 1 - y \leq 0$	$y \to 1$	$x \to 1$

$$\neg(-x + z + 1 \leq 0) \vee \neg(x - y \leq 0) \vee z + 1 - y \leq 0$$

$$-x + z + 1 \leq 0, \quad x - y \leq 0 \qquad\qquad z \to 0, \quad y \to 1$$

$$\equiv$$

$$z + 1 \leq x, \quad x \leq y$$

$$1 \leq x, \quad x \leq 1$$

MCSat – Finite Basis

Every theory that admits quantifier elimination has a finite basis (given a fixed assignment order)

$$F[x, y_1, \dots, y_m]$$

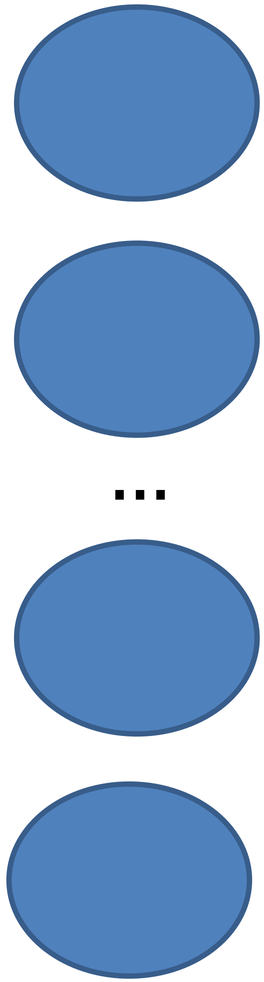
$$y_1 \to \alpha_1, \dots, y_m \to \alpha_m$$

$$\exists x: F[x, y_1, \dots, y_m]$$

$$C_1[y_1, \dots, y_m] \wedge \cdots \wedge C_k[y_1, \dots, y_m]$$

$$\neg F[x, y_1, \dots, y_m] \vee C_k[y_1, \dots, y_m]$$

MCSat – Finite Basis



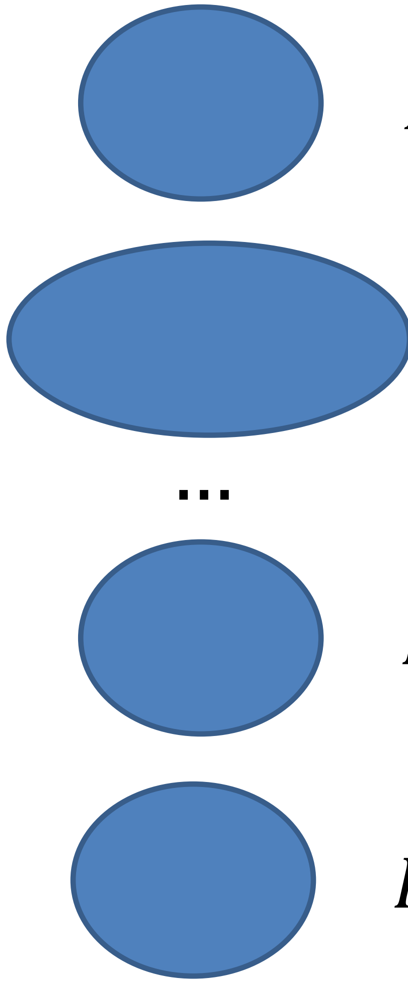
$F_n[x_1, x_2, \ldots, x_{n-1}, x_n]$

$F_{n-1}[x_1, x_2, \ldots, x_{n-1}]$

$F_2[x_1, x_2]$

$F_1[x_1]$

MCSat – Finite Basis



$$F_n[x_1, x_2, \ldots, x_{n-1}, x_n]$$

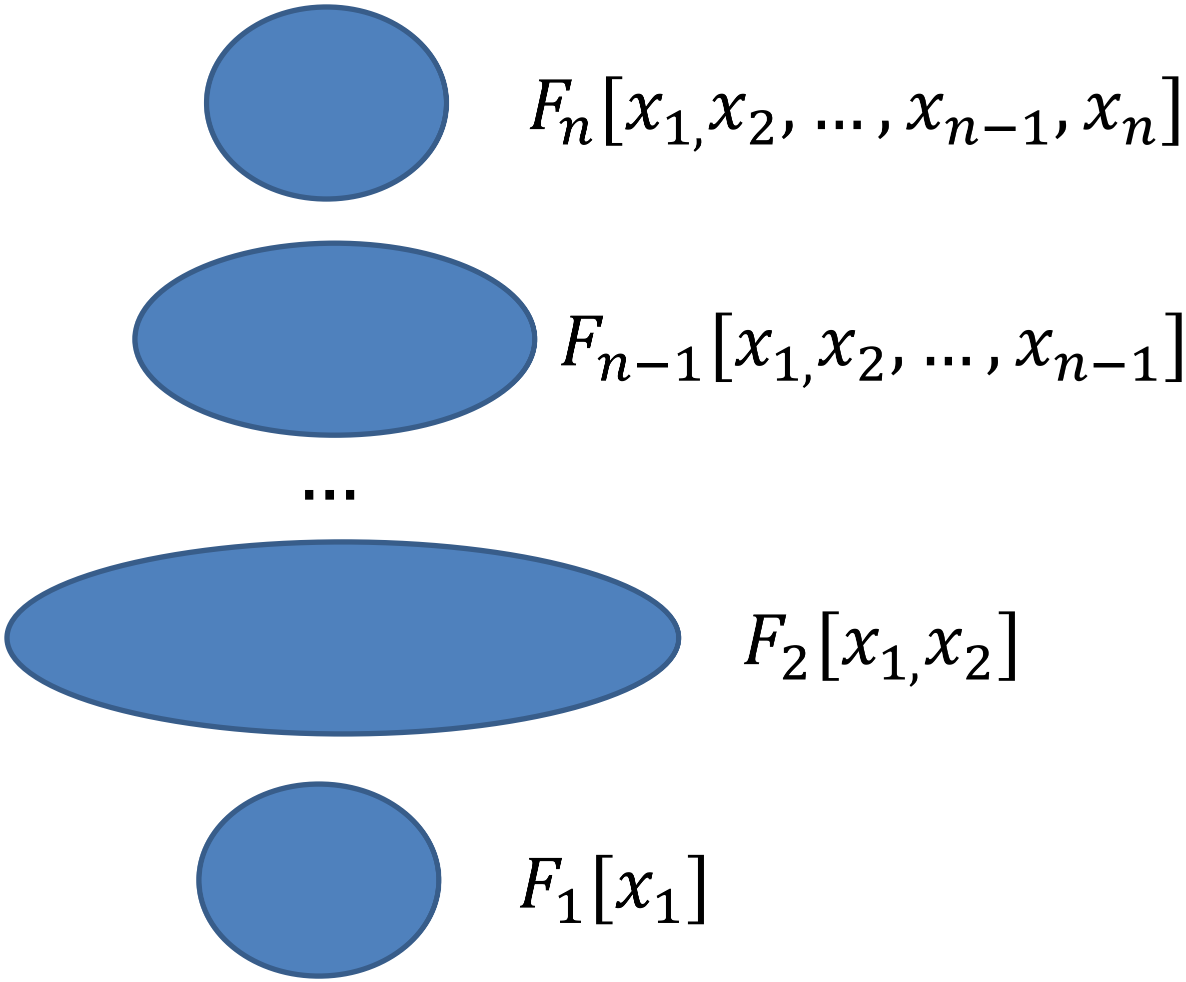
$$F_{n-1}[x_1, x_2, \ldots, x_{n-1}]$$

…

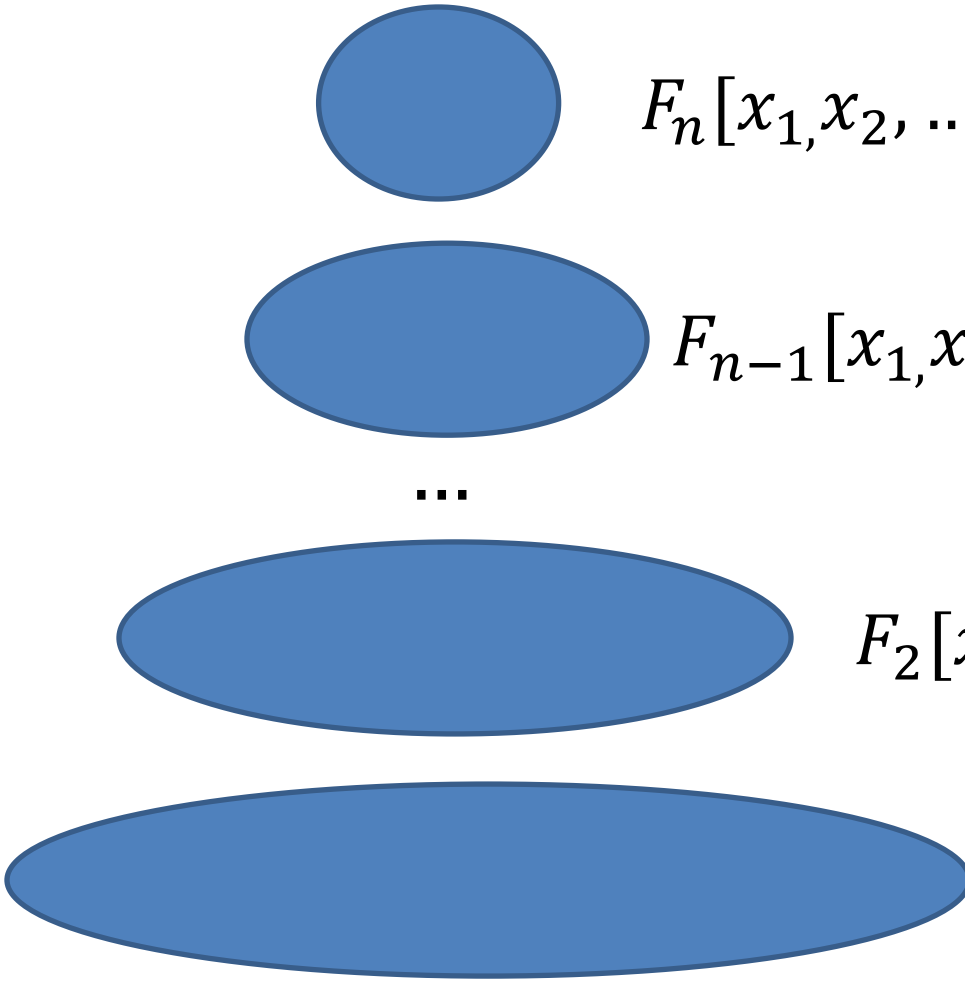
$$F_2[x_1, x_2]$$

$$F_1[x_1]$$

MCSat – Finite Basis



MCSat – Finite Basis



$F_n[x_1, x_2, \ldots, x_{n-1}, x_n]$

$F_{n-1}[x_1, x_2, \ldots, x_{n-1}]$

…

$F_2[x_1, x_2]$

$F_1[x_1]$

MCSat – Finite Basis

Every "finite" theory has a finite basis
Example: Fixed size Bit-vectors

$F[x, y_1, \ldots, y_m]$

$y_1 \to \alpha_1, \ldots, y_m \to \alpha_m$

$$\neg F[x, y_1, \ldots, y_m] \lor \neg(y_1 = \alpha_1) \lor \cdots \lor \neg(y_m = \alpha_m)$$

MCSat – Finite Basis

Theory of uninterpreted functions has a finite basis

Theory of arrays has a finite basis [Brummayer- Biere 2009]

In both cases the Finite Basis is essentially composed of equalities between existing terms.

MCSat: Uninterpreted Functions

$a = b + 1, f(a-1) < c, f(b) > a$

$a = b + 1, f(k) < c, f(b) > a, k = a - 1$

$a = b + 1, f(k) < c, f(b) > a, k = a - 1$

Treat $f(k)$ and $f(b)$ as variables

Generalized variables

MCSat: Uninterpreted Functions

$a = b + 1, f(k) < c, f(b) > a, k = a - 1$

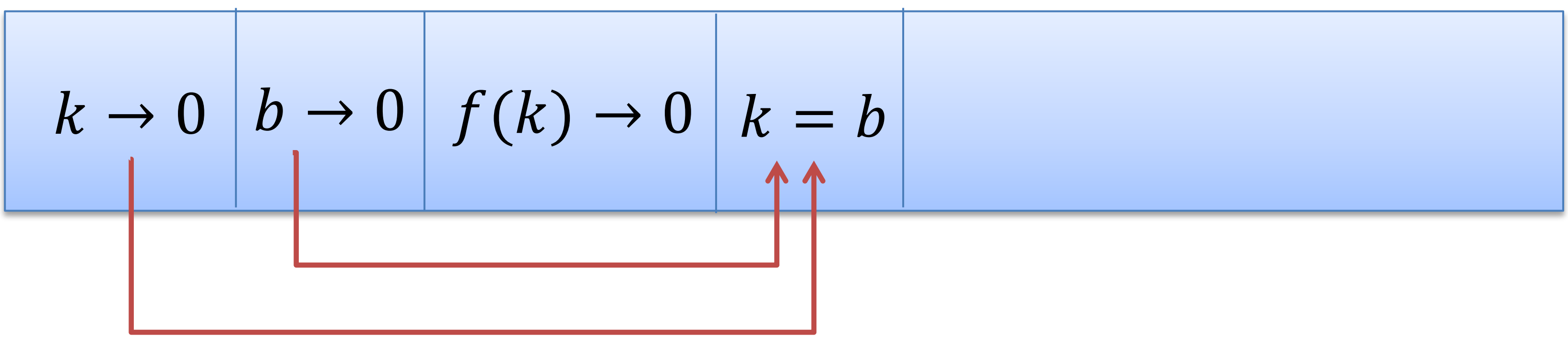
$k \to 0$	$b \to 0$	$f(k) \to 0$	$f(b) \to 2$	

Conflict: $f(k)$ and $f(b)$ must be equal

$\neg(k = b) \vee f(k) = f(b)$

MCSat: Uninterpreted Functions

$$a = b + 1, f(k) < c, f(b) > a, k = a - 1$$

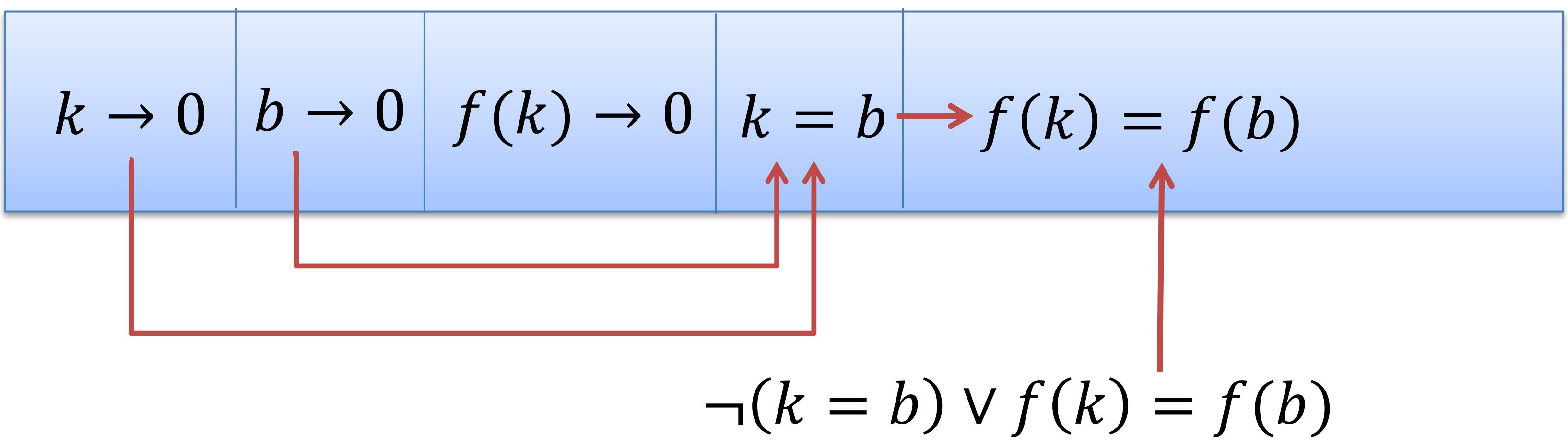


(Semantic) Propagation

$$\neg(k = b) \vee f(k) = f(b)$$

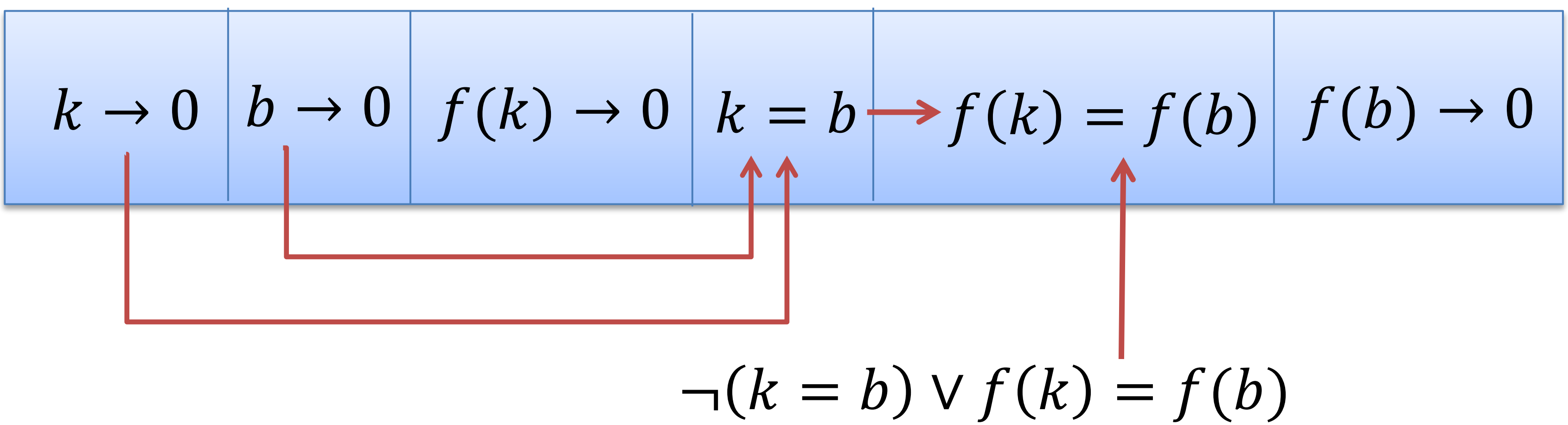
MCSat: Uninterpreted Functions

$$a = b + 1, f(k) < c, f(b) > a, k = a - 1$$



MCSat: Uninterpreted Functions

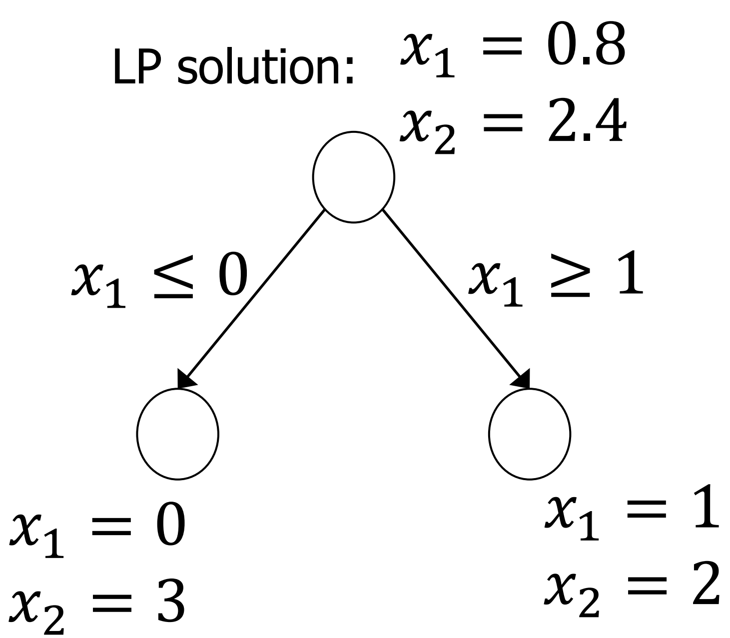
$$a = b + 1, f(k) < c, f(b) > a, k = a - 1$$

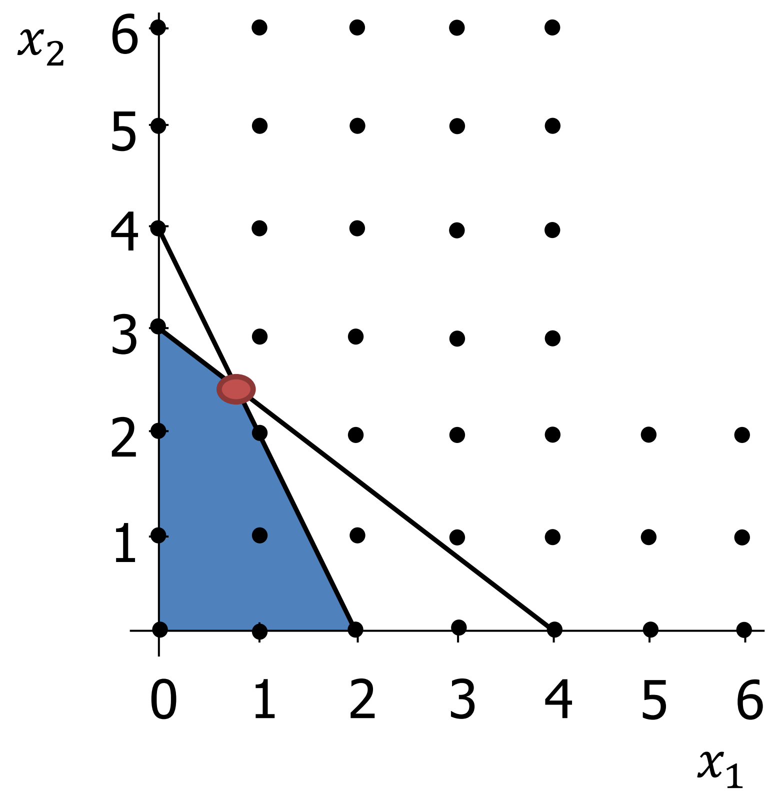


MCSat – Finite Basis

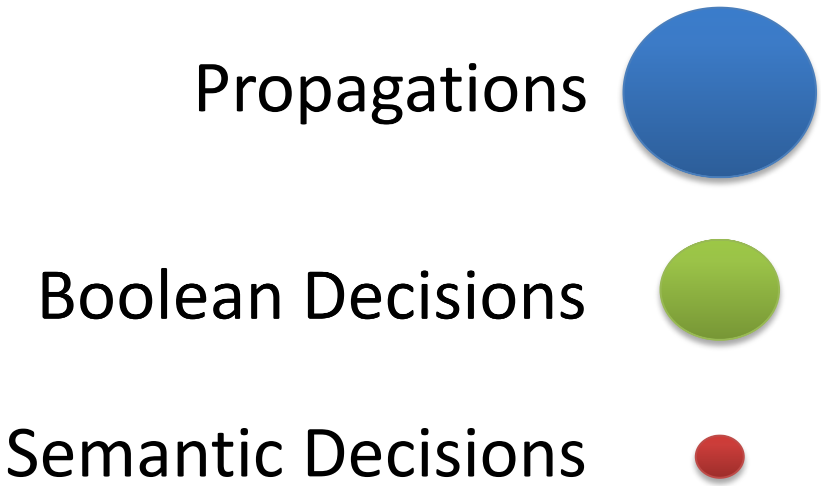
We can also use literals from the finite basis in decisions.

Application: simulate branch&bound for bounded linear integer arithmetic

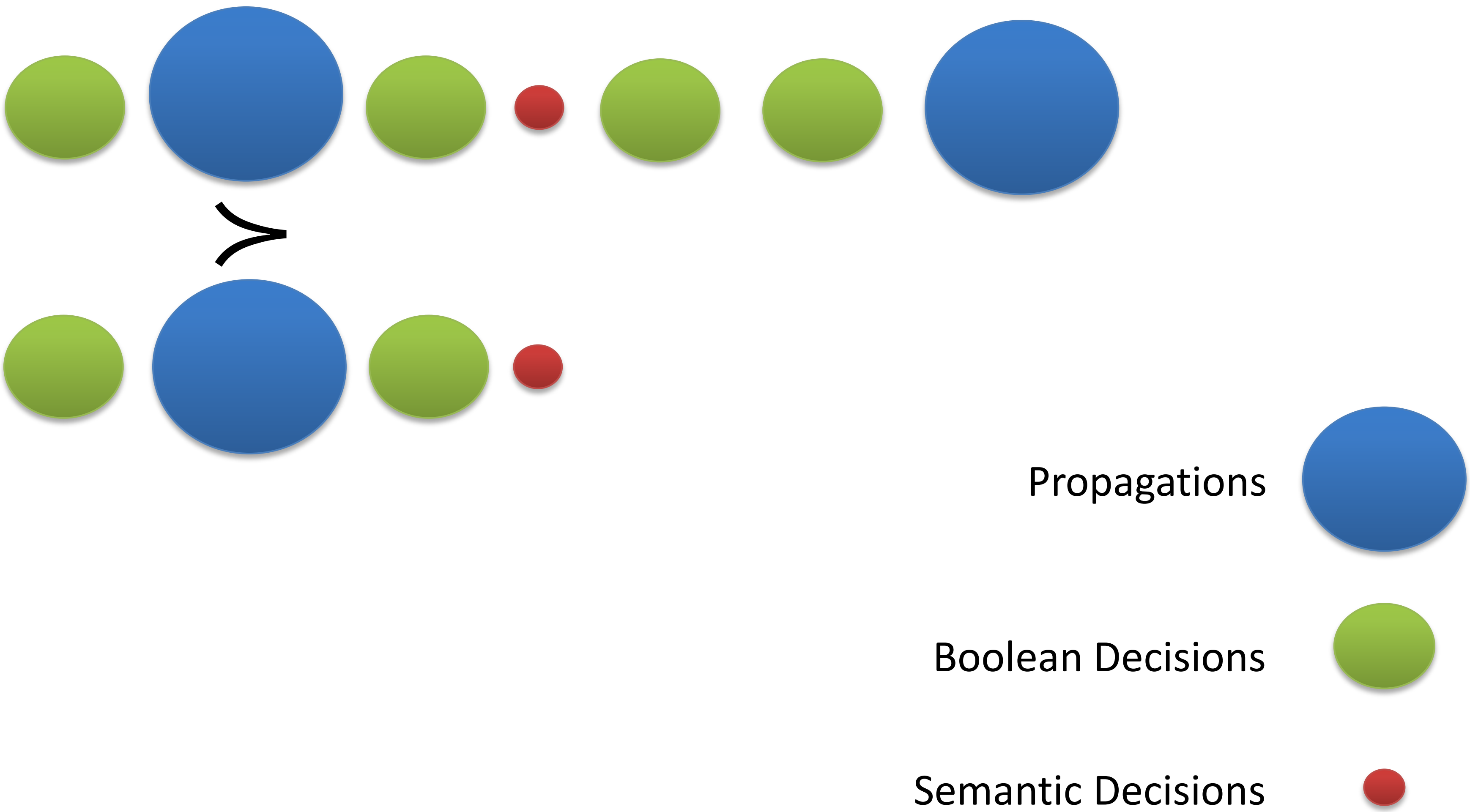




MCSat: Termination



MCSat



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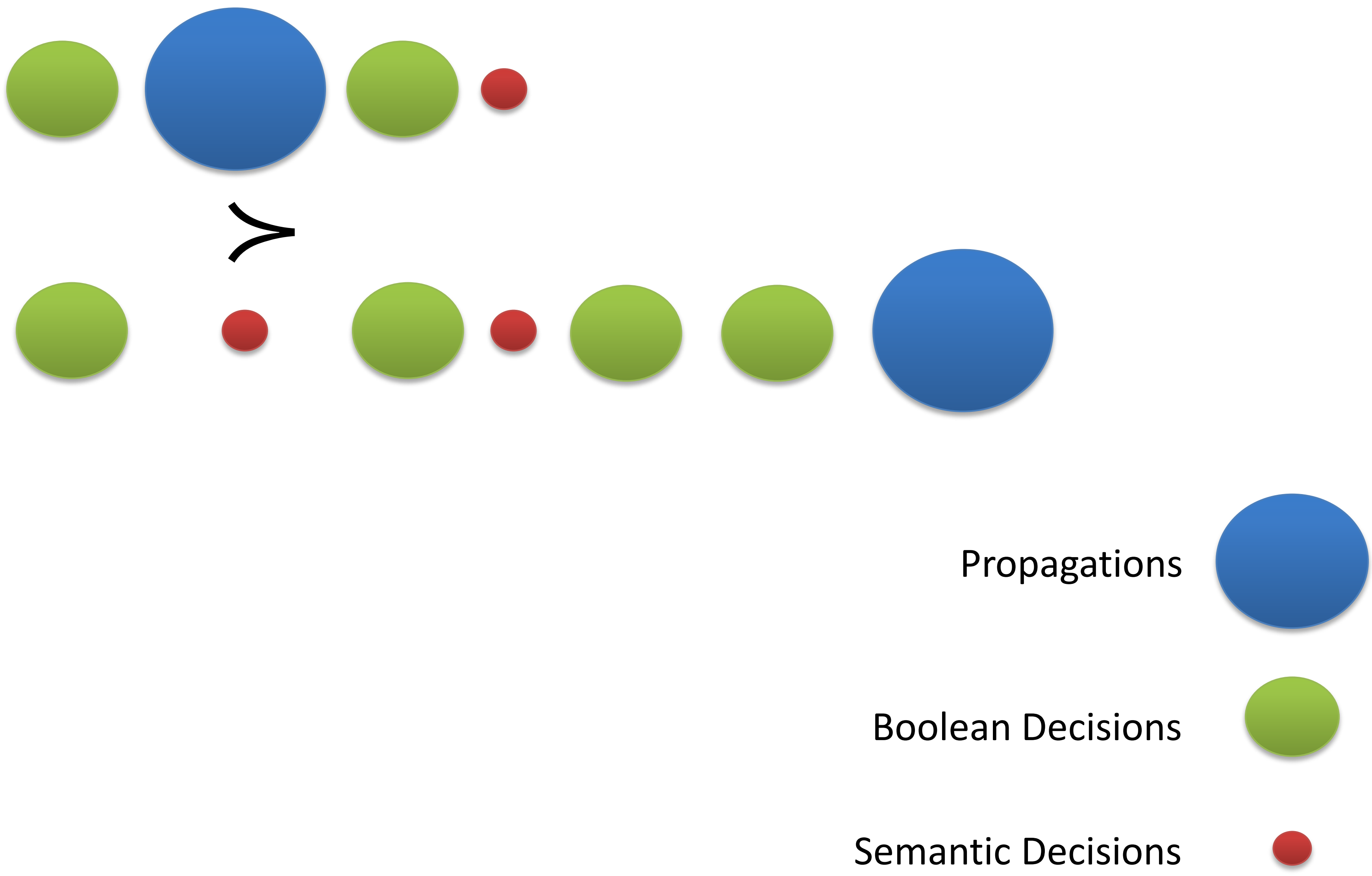
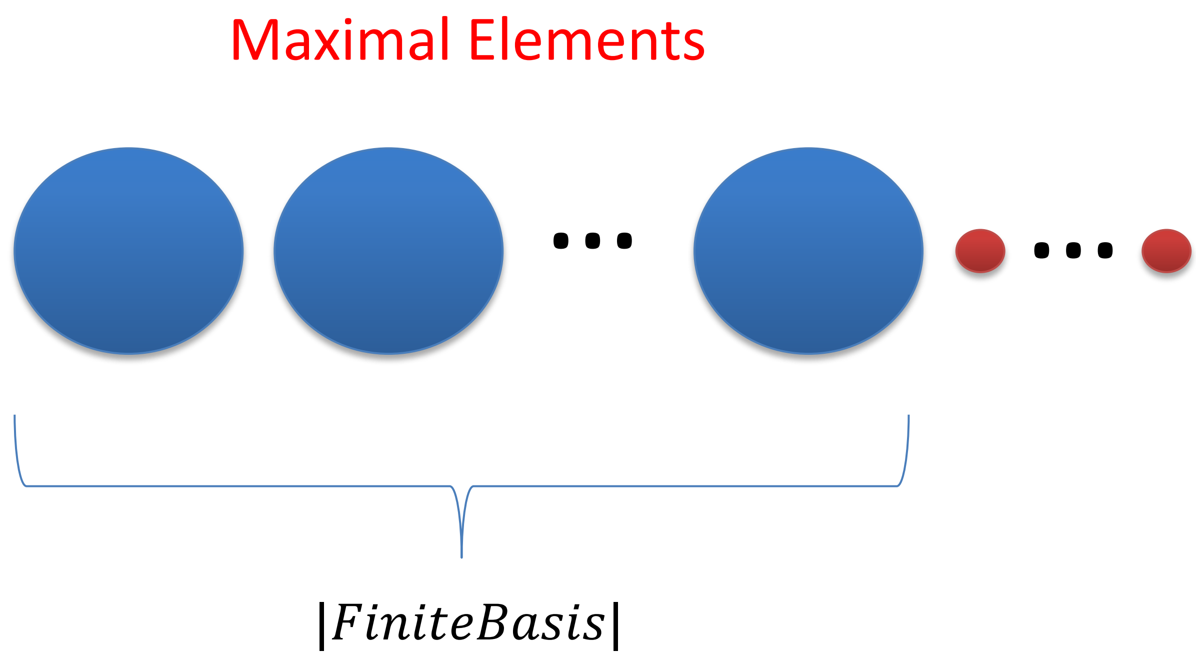
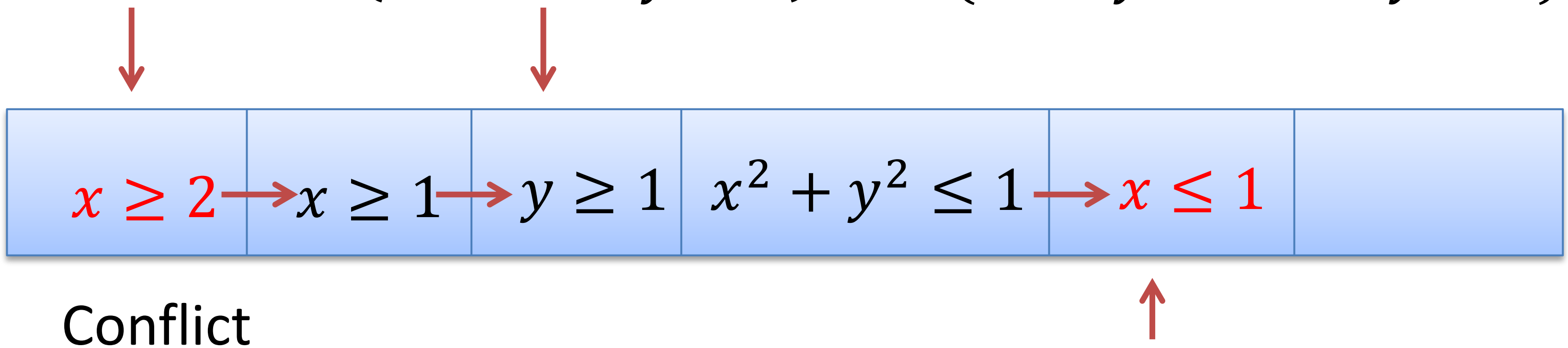


MCSat



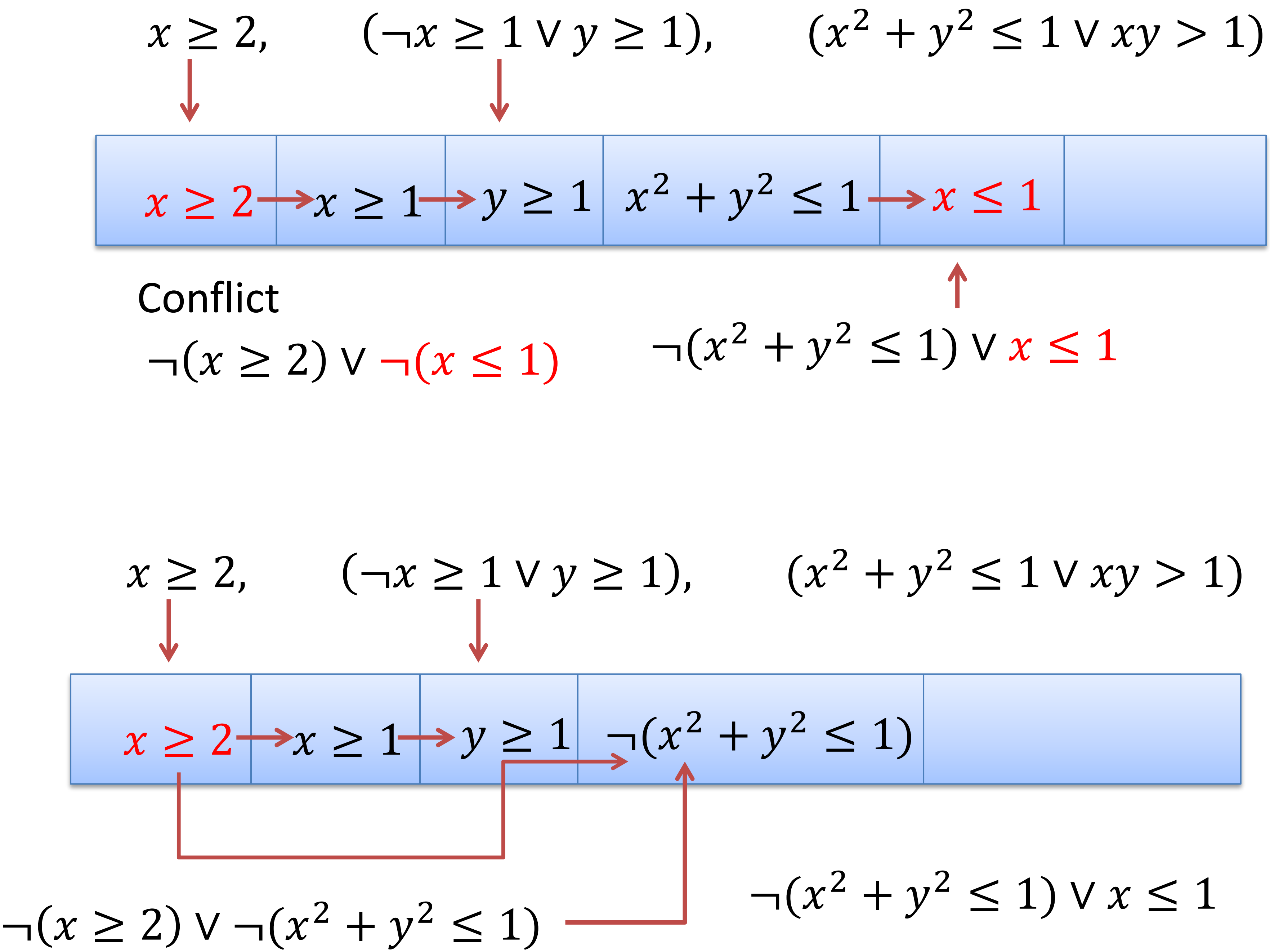
$x \geq 2, \quad (\neg x \geq 1 \vee y \geq 1), \quad (x^2 + y^2 \leq 1 \vee xy > 1)$

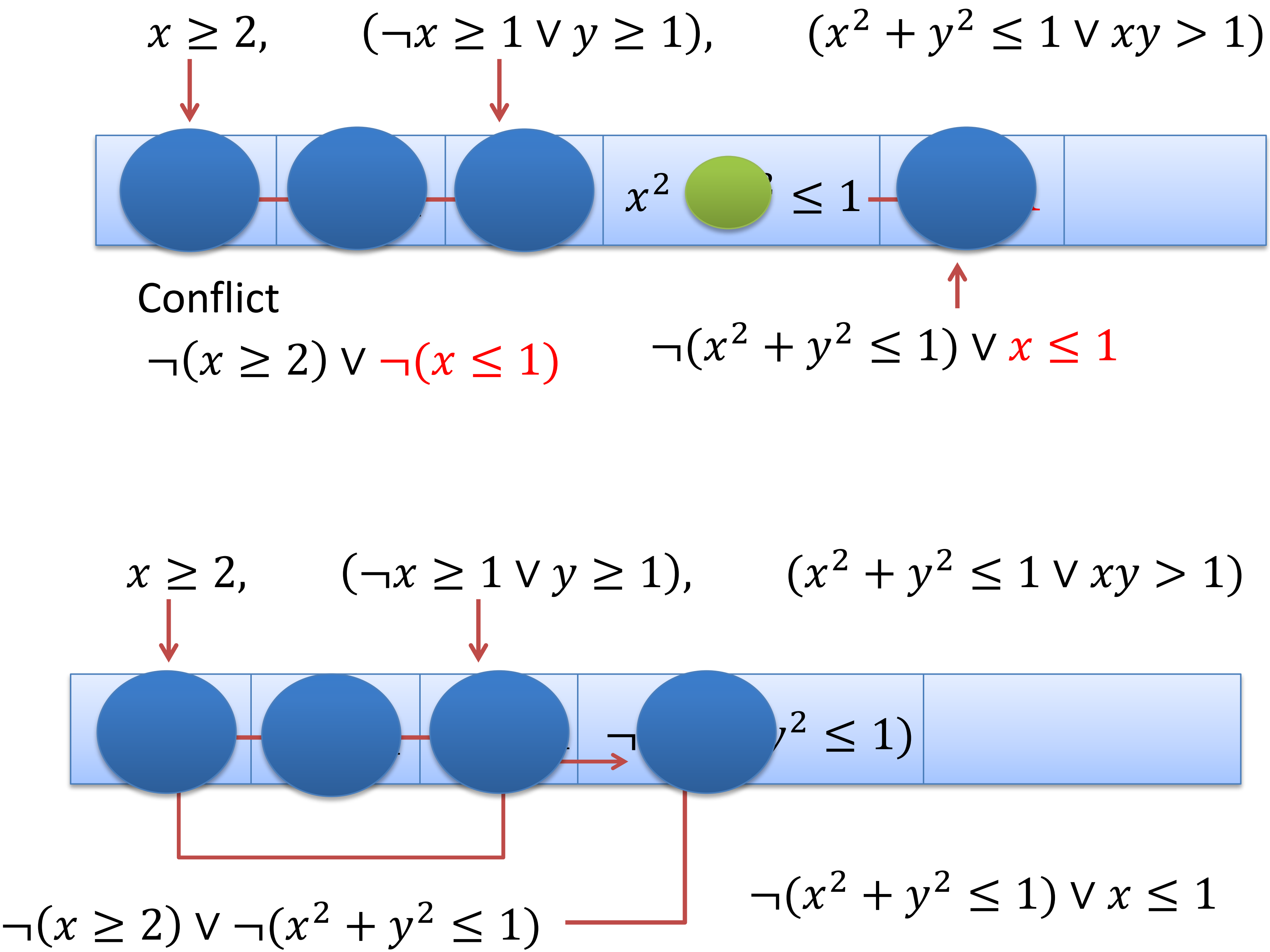


Conflict

$\neg(x \geq 2) \vee \neg(x \leq 1)$

$\neg(x^2 + y^2 \leq 1) \vee x \leq 1$


x ≥ 2,
(¬x ≥ 1 ∨ y ≥ 1),
(x² + y² ≤ 1 ∨ xy > 1)
x ≥ 2
x ≥ 1
y ≥ 1
x² + y² ≤ 1
x ≤ 1
Conflict
¬(x ≥ 2) ∨ ¬(x ≤ 1)
¬(x² + y² ≤ 1) ∨ x ≤ 1
x ≥ 2,
(¬x ≥ 1 ∨ y ≥ 1),
(x² + y² ≤ 1 ∨ xy > 1)
x ≥ 2
x ≥ 1
y ≥ 1
¬(x² + y² ≤ 1)
¬(x ≥ 2) ∨ ¬(x² + y² ≤ 1)
¬(x² + y² ≤ 1) ∨ x ≤ 1


$x \geq 2,$
$(\neg x \geq 1 \vee y \geq 1),$
$(x^2 + y^2 \leq 1 \vee xy > 1)$
Conflict
$\neg(x \geq 2) \vee \neg(x \leq 1)$
$\neg(x^2 + y^2 \leq 1) \vee x \leq 1$
$x \geq 2,$
$(\neg x \geq 1 \vee y \geq 1),$
$(x^2 + y^2 \leq 1 \vee xy > 1)$
$\neg(x \geq 2) \vee \neg(x^2 + y^2 \leq 1)$
$\neg(x^2 + y^2 \leq 1) \vee x \leq 1$

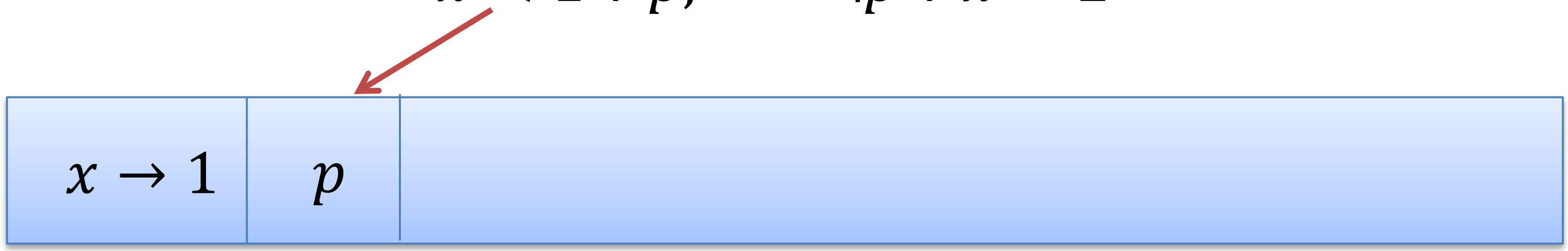
MCSat

$$x < 1 \lor p, \qquad \neg p \lor x = 2$$

$x \to 1$	

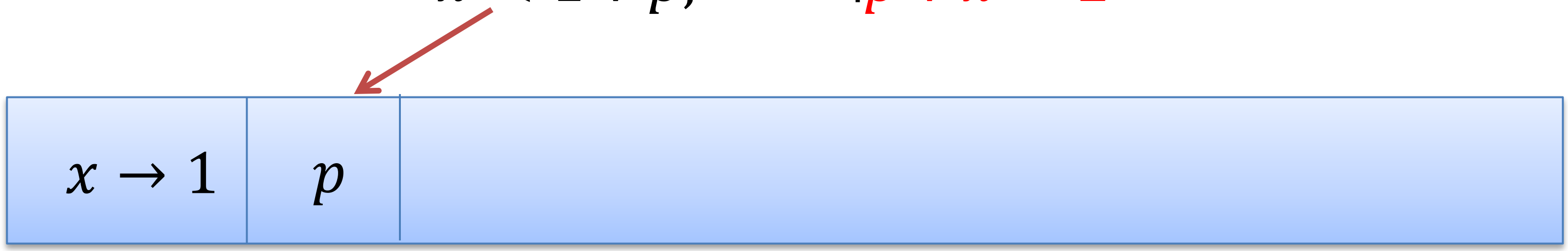
MCSat

$x < 1 \vee p, \qquad \neg p \vee x = 2$



MCSat

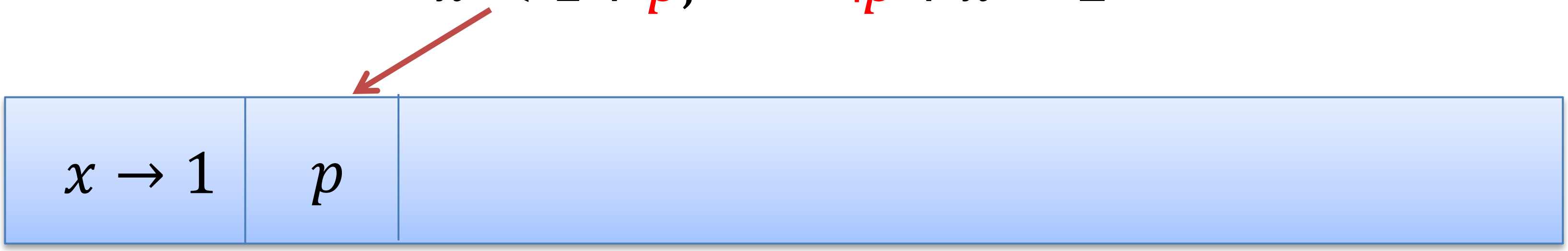
$x < 1 \vee p, \qquad \neg p \vee x = 2$



Conflict (evaluates to false)

MCSat

$x < 1 \vee {\color{red}p}, \qquad {\color{red}\neg p} \vee x = 2$

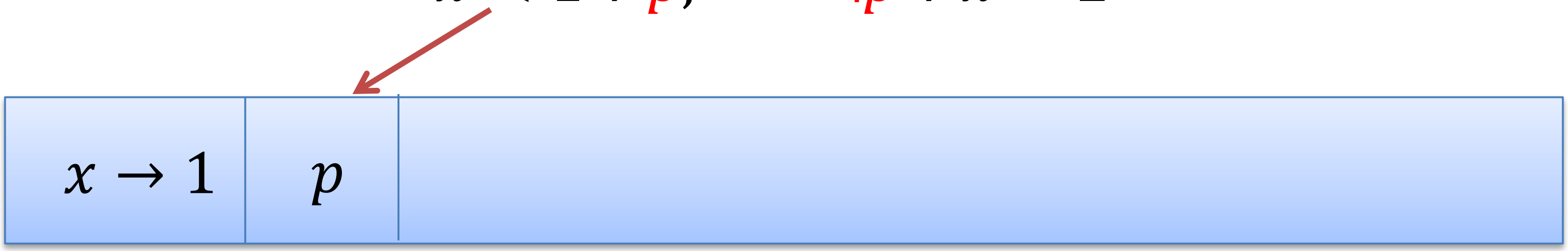


New clause

$x < 1 \vee x = 2$

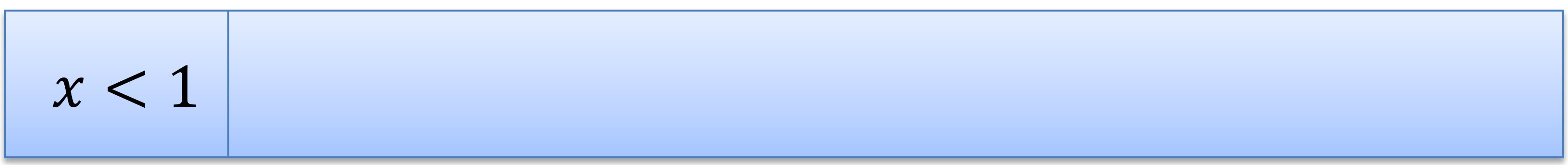
MCSat

$x < 1 \vee p, \qquad \neg p \vee x = 2$



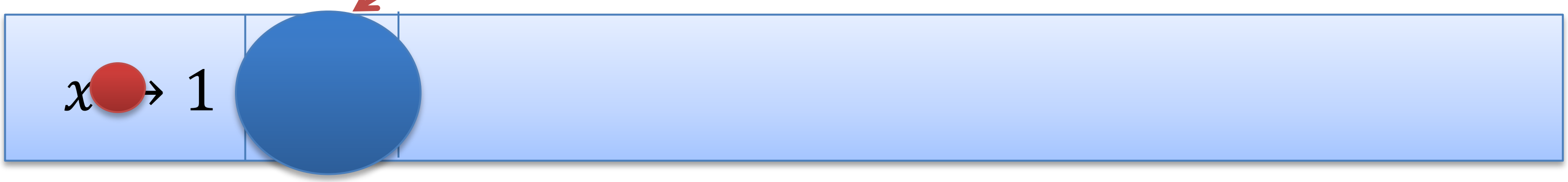
New clause

$x < 1 \vee x = 2$



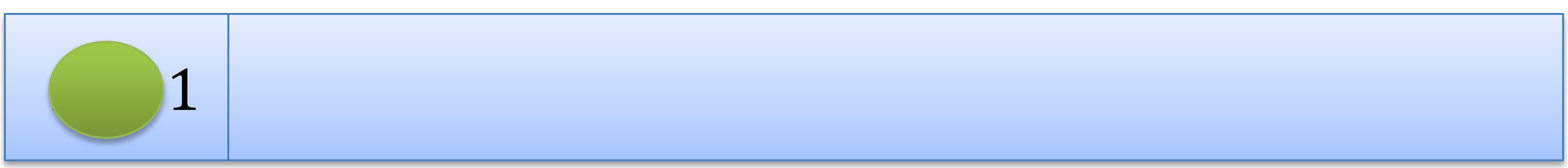
MCSat

$x < 1 \vee p, \quad \neg p \vee x = 2$

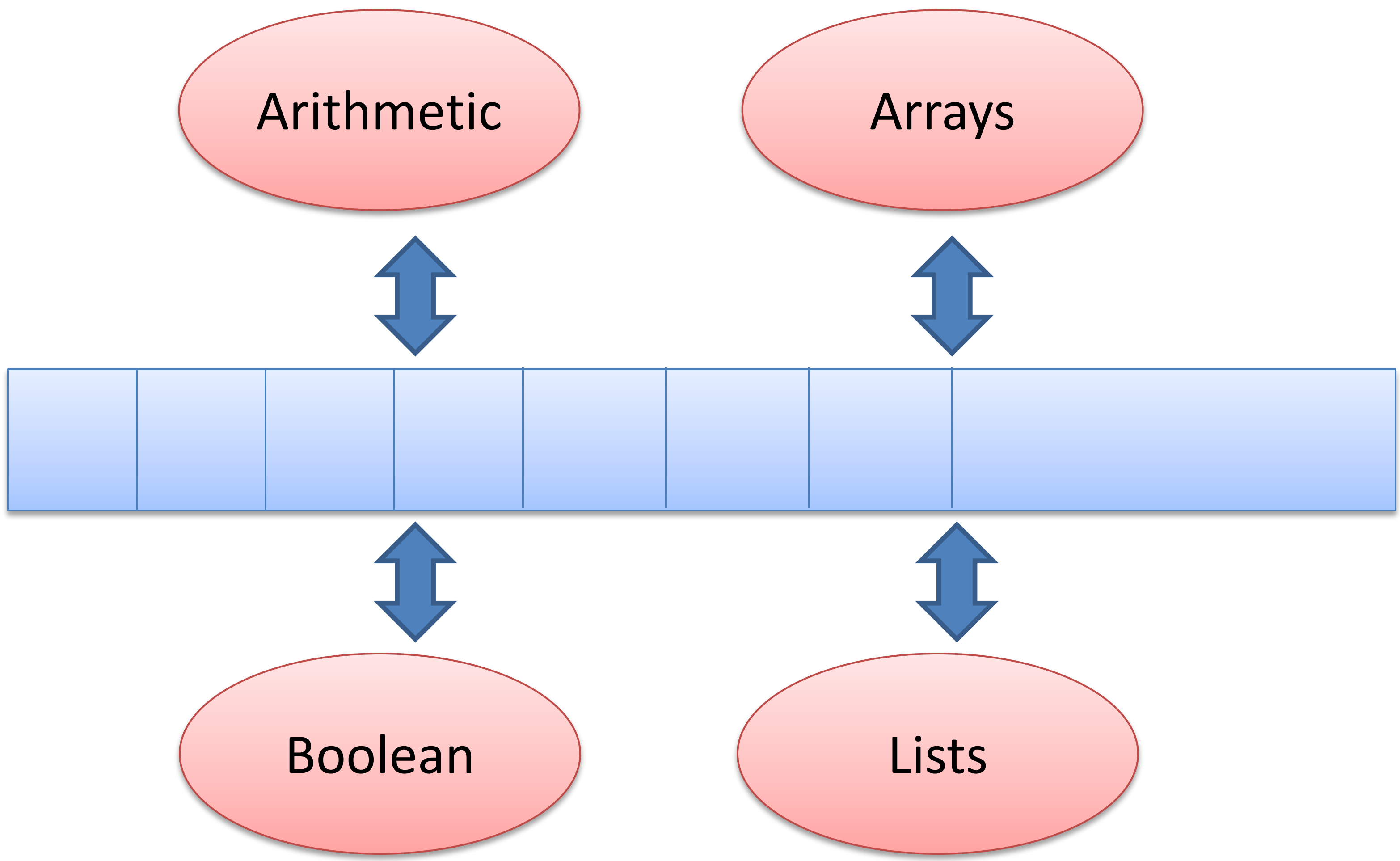


New clause

$x < 1 \vee x = 2$



MCSat: Architecture



MCSat prototype: 7k lines of code

Deduction Rules

$$\frac{C \vee L \qquad \neg L \vee D}{C \vee D}$$ Boolean Resolution

$$\frac{}{\neg(p_L < x) \vee \neg(x < p_U) \vee (p_L < p_U)}$$ Fourier-Motzkin

$$\frac{}{(p = q) \vee (q < p) \vee (p < q)}$$ Equality Split

$$\frac{}{x_1 \neq y_1 \vee \cdots \vee x_k \neq y_k \vee f(x_1, \ldots, x_k) = f(y_1, \ldots, y_k)}$$ Ackermann expansion aka Congruence

$$\frac{\neg(p < q) \vee x \vee x}{(q \leq p) \vee x}$$ Normalization

MCSat: preliminary results

prototype: 7k lines of code

QF_LRA

	mcsat		cvc4		z3		mathsat5		yices	
set	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)
clocksynchro (36)	**36**	**123.11**	36	1166.55	36	1828.74	36	1732.59	36	1093.80
DTPScheduling (91)	**91**	**31.33**	91	72.92	91	100.55	89	1980.96	91	926.22
miplib (42)	8	97.16	**27**	**3359.40**	23	3307.92	19	5447.46	23	466.44
sal (107)	107	12.68	107	13.46	107	6.37	107	7.99	**107**	**2.45**
sc (144)	144	1655.06	144	1389.72	144	954.42	144	880.27	**144**	**401.64**
spiderbenchmarks (42)	42	2.38	42	2.47	42	1.66	42	1.22	**42**	**0.44**
TM (25)	25	1125.21	25	82.12	**25**	**51.64**	25	1142.98	25	55.32
ttastartup (72)	70	4443.72	72	1305.93	72	1647.94	72	2607.49	**72**	**1218.68**
uart (73)	73	5244.70	73	1439.89	73	1379.90	73	1481.86	**73**	**679.54**
	596	12735.35	**617**	**8832.46**	613	9279.14	607	15282.82	613	4844.53

MCSat: preliminary results

prototype: 7k lines of code

QF_UFLRA and QF_UFLIA

	mcsat		cvc4		z3		mathsat5		yices	
set	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)
EufLaArithmetic (33)	33	39.57	33	49.11	**33**	**2.53**	33	20.18	33	4.61
Hash (198)	198	34.81	198	10.60	198	7.18	198	1330.88	**198**	**2.64**
RandomCoupled (400)	400	68.04	400	35.90	400	31.44	**400**	**18.56**	384	39903.78
RandomDecoupled (500)	500	34.95	500	40.63	500	30.98	**500**	**21.86**	500	3863.79
Wisa (223)	223	9.18	223	87.35	223	10.80	223	65.27	**223**	**2.80**
wisas (108)	**108**	**40.17**	108	5221.37	108	443.36	106	1737.41	108	736.98
	1462	**226.72**	1462	5444.96	1462	526.29	1460	3194.16	1446	44514.60

Conclusion

Logic as a Service

Model-Based techniques are very promising

MCSat

http://z3.codeplex.com

http://rise4fun.com/z3